Saari, with no Apologies

Deanna Haunsperger

Deanna Haunsperger (dhaunspe@carleton.edu; Carleton College, Northfield, MN 55057) received her BA from Simpson College and in 1991 her PhD from Northwestern University. She taught for three years at St. Olaf College before arriving at Carleton, where she is now an Associate Professor of Mathematics. She was co-Editor of *Math Horizons* from 1999 to 2003, and she has directed the Carleton Summer Mathematics Program for women since 1995. In her free time she enjoys spending time with her family.

You might expect a member of the National Academy of Sciences, Editor of the *Bulletin of the American Mathematical Society*, author of well over a hundred publications, winner of the Ford, the Allendorfer, and the Chauvenet Prizes for expository writing, Guggenheim Fellow, and author of the books *The Geometry of Voting* and *Chaotic Elections! A Mathematician Looks at Voting* to be too busy to discuss the importance of mathematics with a group of non-mathematicians or a class of fourth-graders. But if you have read what Donald G. Saari, Distinguished Professor and Director of the Institute for Mathematical Behavioral Sciences at the University of California, Irvine, has written, or if you have had the pleasure of a conversation with this consummate story-teller, you would find you were wrong. Proselytizing the beauty, power, and ubiquity of mathematics is not an obligation to Saari—it's his passion. One afternoon at Mathfest 2003 I had the opportunity to talk with him about growing up in Michigan, his education, and his mathematics.

Tell me about growing up and becoming a mathematician.

I grew up in the Upper Peninsula of Michigan, in a Finnish-American region right in the midst of the copper mining area. It was like a frontier town, where copper mining was the big industry. We lived on the edge of town, so as a kid I spent quite a bit of time in the woods, hunting, camping, snow-shoeing, fishing, and even climbing down and exploring abandoned copper mines for mineral samples. I was involved in Boy Scouts, in athletics, in forensics, in class plays. You name it, I was involved in it.

My parents were idealists. My mother graduated as valedictorian of her class at sixteen; by the time she was seventeen, the people in the local churches were campaigning against her because she believed in, of all things, women's rights. Things that today we would accept as absolutely trivial were not taken lightly at that time in some of the conservative areas. My father also was very much of an idealist. He spent time in jail for leading a protest against the fact that farmers were losing their farms during the Great Depression. Together they decided to channel their ideas and energy in the labor movement. They were organizers and community activists in the area, which meant there was a lot of idealism in our family, many inspiring discussions, but no money. Never any money. But who needed money?

Grade school and high school were very, very delightful. Perhaps I enjoyed people a little too much, as I would gab and joke even during class time. As a result I served

Figure 1. Even in the 1940s, Saari was outstanding in his field.

more than my share of detentions. The teacher in charge of detentions was Bill Brotherton, our algebra teacher. He was always delighted when he heard that I was headed for the after-school detention session because he would bring in all sorts of math books. I was essentially getting a free math tutorial on different aspects of math that weren't taught in my school at that time. He was very good, very instrumental in developing my interest in mathematics.

I don't recall what I planned to do when I grew up. I always enjoyed academics, but I enjoyed athletics, I enjoyed acting, I enjoyed interacting with people—quite frankly, I always enjoy a good time and bad jokes. I knew I liked mathematics, but at that time I didn't realize you could have a good living, a good life in mathematics. At that time I thought that the only thing you could do with mathematics is be a high school teacher. While being a high school teacher would have been fine, I just wanted to move out of the area and look around.

When I graduated from high school I applied to two schools: one of the Ivies and Michigan Tech. I was accepted to both. I couldn't afford to go to the Ivy; I couldn't even afford to pay the transportation to get there, and Michigan Tech was on the other side of town. The scholarship they gave me paid all my university expenses, so I went to Michigan Tech for financial reasons, but it was an excellent experience. An absolutely excellent experience. In our graduating class we had George Gasper who's now at Northwestern, Dan Maki who's at Indiana, Jim Thomas at Colorado State, and a couple of others. I'd say at least half of us ended up in university professorships.

When I was at Michigan Tech, I had a triple major: social life, athletics, and campus politics. I entered as a typical undergraduate, changing majors as fast as some people change shirts. I started off in chemistry primarily because I had very high grades in the subject, but I hated the labs. Then I changed into electrical engineering because I heard it was one of the more difficult majors, but I was responsible for a couple of explosions in the labs there, too, trying to hurry things through to go out to have coffee. I still recall my instructor, putting his arm around my shoulder and hopefully recommending, "Don, have you considered a non-lab major?" So, I moved into math because I enjoyed it so much and I found it easy. Gene Ortner was my instructor in

Figure 2. Enjoying a nice day outside at Michigan Tech in the 1960s.

my first course in abstract algebra. Oh, was that delightful; the way he taught it made that subject seem so beautiful. And my major just evolved into mathematics because that was my real interest anyway. Always has been, always will be. It was that abstract algebra course, a couple of the other courses, and the dedicated Tech instructors at the time that motivated me to go on to graduate school: involved instructors can have a huge impact on the future life of a student!

I was accepted to several graduate schools, but quite frankly I was not very sophisticated at making the decision about where to go. As I looked through the places where I applied and the offers I had, I thought, well Purdue looks pretty good, so I went to Purdue. I'm pleased with my decision. I had an absolutely tremendous experience at Purdue. I was one of those graduate students who, when I arrived, because of my positive experience in algebra, knew I was going to be an algebraist. Absolutely, there was no question; I was going to be an algebraist. I loved the courses I took, but then I discovered algebraic topology, and is that a beautiful subject! So then I was going to be an algebraic topologist. After that I got hooked by analysis. In complex analysis the powerful results you learn at an early stage are just so beautiful, so I decided to become an analyst. Then I discovered applied mathematics, and that was nice, but I realized I had to understand functional analysis better if I'm going to do partial differential equations. So I started toward a thesis in functional analysis when my advisor said "Don, I know you like teaching. Harry Pollard is considered our best teacher. You won't like the material (celestial mechanics, the n-body problem), but why don't you sit in so you can see how he teaches?" Being a student, I made the obvious bad jokes about how celestial mechanics is the study of how heavenly bodies move, but I agreed, and once I was in the class, I found the material so fascinating and so nice, that I made my next switch to the n-body problem, and I wrote my thesis about collisions in the n-body problem.

Harry Pollard was different. Delightful. We would get together and talk about mathematics maybe once or twice a week. Any branch of mathematics. He never posed a research problem for me or anything else, we would just sit down and talk about mathematics. It was enjoyable and educational, but Harry had no idea what I was doing or if

I was even working. The first he really learned about my research was when I handed him a draft of my thesis. He responded, "Oh, so you are working!"

I was at Purdue five years, but one year was spent courting my wife Lil. Of course you understand, when I finally got enough courage to ask her to marry me, her response was "What? And be 'Saari' for the rest of my life?!"

After Purdue, I went to Yale. I was there only a short time when the phone rang. It was Ralph Boas from Northwestern, and he said "I understand you're looking for a job." Well, I wasn't, but I said "Sure!" So he invited me out and I gave a "job-talk" lecture on the collision orbits in the Newtonian n-body problem. What attracted me to Northwestern was that some of the techniques I was using in the n-body problem, both in the work I did in the collision orbits and in my later work describing the evolution of the n-body problem (that is, given any number of particles, explaining how the system evolves as time goes to infinity), I used an analytic tool called Non-linear Tauberian theorems. They should be called Hardy-Littlewood theorems because they're the ones who extracted them from theorems that had been done earlier by Tauber. To give a little history, Wintner at Johns Hopkins University in the early thirties asked Ralph Boas to look at a result by this Finnish mathematician by the name of Sundman. Wintner suggested that there was a Tauberian argument hidden in Sundman's analytic argument about two-body collision orbits. Ralph worked on it and, indeed, extracted a Tauberian theorem out of it. In separate papers, Harry and I generalized Ralph's insights into a non-linear Tauberian theorem that became one of the tools I used extensively in my work on the Newtonian n-body problem. Ralph was at Northwestern and I was an admirer of his work, as well as that of Avner Friedman, Bob Williams, and a large number of other people, so there were excellent reasons for me to be interested in a job at Northwestern. When they offered me a position, I took it instantly; we moved to Evanston, where we remained for 32 years.

You have a wide variety of mathematical interests. You're published in Celestial Mechanics, Dynamical Systems, Social Choice, Mathematical Economics, Game Theory, Voting Theory, Mathematical Psychology. How did you get interested in so many things? What attracts you to a problem?

I was in my first year at Northwestern teaching a graduate-level course in functional analysis. I was asked to let my colleagues know who our best students were. When I said Jim Jordan, they said, "He's not one of ours." So I identified the second best by name. "Well, he's not one of ours either." Well, the third best is so-and-so. "She's not a math major either." Something strange was going on, so I asked these students, "What are you doing in here beating up on our poor math majors?" They were graduate students in economics. At that time I probably thought that economists, I don't know, looked at the stock market or something; I discovered that when they examined economies with a large number of commodities or economies with a large number of agents, in a natural manner the issues could be posed as problems in functional analysis. I became intrigued by what they were doing. A couple of years later, John Ledyard in the Economics Department and I ran a seminar in the Infinite Dimensional Economy where I would worry about the mathematics and he would worry about the economics. The more I learned, the more it became clear that there were some absolutely fascinating issues over there.

Another source of my interest came from Hugo Sonnenschein, who left Northwestern for Princeton, then became provost at Princeton, then president of the University of Chicago. Hugo proved the very interesting theorem that in the usual Adam Smith story of supply and demand, rather than behaving nicely, the economy could do anything. In modern terms, this meant that the supply and demand story could be as chaotic as

It is encouraging that there are many amateur mathematicians, but at times this can be a serious time drain. In the 1970s, during the Cold War, a short guy in a sports coat and claiming to be an engineer asked if he could talk with me. When he wanted to close my door, I became suspicious. "Why?" I asked. His response, "I don't want the Russians to know what I am going to say."

Standing by my blackboard with a proud smile, he made his big announcement, "What would you say if I told you that I solved Fermat's Last Theorem?" Thinking quickly, I responded: "Nothing. This is because our department chair divides up all of the unsolved problems among the faculty, and I was not assigned that problem. Therefore I'm not allowed to even look at it." After I answered his obvious next question by identifying "who had been assigned Fermat's Last Problem," my colleague Len Evans spent the next three days first finding the error in his proof, and then convincing him that there was an error.

you'd like it to be. This was difficult to believe. So I became intrigued, trying to figure out if it could be generalized. My generalization proves that this is a very robust result.

I work on whatever intrigues me at the moment. I'm writing a book right now on the n-body problem with some new results. I am just finishing up a paper on mathematical psychology on how individuals make decisions. The work of Duncan Luce has been very important in this area. He received his PhD in mathematics from MIT. Then he and Raiffa wrote a book that had, and still has, a big impact on game theory. Next Duncan became interested in mathematical psychology, and he is one of the founders of this field. I started reading some of Luce's work, and found that it was beautiful. My paper in this area extends some of his work. Quite frankly, the social sciences are becoming more mathematically sophisticated. If you attend a mathematical economics conference, you'll hear talks where they're using various aspects of topology or functional analysis, and they're very comfortable with all of these different concepts. This diversity is not true with the other social sciences, but even these other areas are becoming more mathematically sophisticated as time goes on. Attending these conferences, you begin to discover some excellent problems, excellent issues, that, while the social scientists are very good, they don't always understand all of the mathematics or even what is possible to do with mathematics. This is where mathematicians can make a tremendous contribution to these areas.

For me, a problem is interesting if you really don't understand the underlying structure—where there are no clues about what's going on. For example, when I started working on the evolution of the n-body problem, we knew Newton's two-body solution and the work of the French mathematician Chazy in the 1920s for the three-body problem. But after that there was no clue, no idea, what would happen in general for the four-body problem, the five-body problem, the n-body problem. The same was true with collisions. We knew what happened if two particles collide or three particles collide, but multiple collisions, or collisions that happened with a five-body collision over here and a three-body collision over there?! How likely are they? Since there was no known structure, no guidelines, I just found the challenge to be absolutely irresistible.

In voting theory, only a couple of paradoxical results were known about what could happen in tallying elections. I thought there should be a way to find all possible paradoxes. So I became intrigued by that question. It is interesting how, by borrowing and modifying notions from chaotic dynamics, I was able to characterize all possible paradoxes that could ever occur. Then, to explain why all of them do occur, I used orbits of symmetry groups.

What doesn't interest me is when the overall structure of a problem is basically known, and it's primarily technical details that need to be worked out. Once the overall structure is understood, I lose interest in the problem. Sometimes I lose interest so much in the problem it takes me a couple of years to get around to writing up results, if ever! Bad strategy because often there are many interesting results just waiting to be extracted once the structure is understood. For instance, I have been lecturing on a result in qualitative evolutionary game theory for over three years, but I have yet to write it up.

What do you think makes a mathematician successful?

I really don't know. Hard work, after all, mathematics is difficult. And you have to enjoy the mathematics. You have to say "I can't wait until I start again tomorrow." I'm not interested in a problem because it is known or technically difficult; for me it has to be important for our understanding of a field. But for me, I think what creates this drive to do mathematics is curiosity. You just plain must be curious. But curiosity becomes almost a demon. It can take over what's going on in your life. Earlier I joked about when I proposed to my wife; let me add here that she is very important to me. I think for any successful mathematician, the spouse deserves considerable credit. Just imagine: you're sitting down in the middle of a nice, romantic dinner—and then, instead of sweet nothings, a spaced-out look comes over you. That's because all of a sudden you see the right mathematical relationship that has been bugging you for the last few months. Wow! With something like that, the only way to keep the marriage strong is to have an incredibly understanding wife! I try to be polite when this happens and snap out of it, but I do scribble on the napkins.

What of your mathematical work do you like best?

I really like my work on the Newtonian n-body problem—the work on collisions, the work on the evolution, the work on restrictions on motion, various things like that. The reason I like my work in that direction is that it was very different from what had been done at that time, and it speaks to what I found to be central issues of how the universe evolves, and so forth. I'm proud of my work in voting theory; again, it was a very different direction from what had been done at that time; it answered a lot of questions. What I also liked is that my work goes beyond the traditional approach of finding negative results (such as stating what is flawed or impossible), to show how to find positive results, such as a new interpretation of Arrow's Impossibility Theorem showing that it does not mean what Arrow thought it meant. My work in mathematical economics I like because it finally convinced me that certain assertions I did not previously believe really were true. Also, this work got me back to non-linear functional analysis and foliation theory, and so the mathematics is just fun.

It sounds like you could never be happy in just one field.

Probably not; in fact, that's one of the reasons that I moved to the University of California, Irvine. I met Duncan Luce, whom I mentioned earlier. Duncan invited me to come out and spend a quarter. Well, I thought I'd go out and see what's going on, learn some of the new mathematical ideas that were being developed there, and then return to Northwestern.

Well, I ran into many excellent people, and they started recruiting me. My initial response was that I wasn't interested. I mean Northwestern has always been very, very good to me, and I had a tremendous time at Northwestern. My ties to Northwestern were so tight and so strong and I had such a positive experience there, that the idea of leaving was totally out of the question. My daughter, Anneli and her husband were thinking about buying a home near us in Evanston, so they asked "Dad, I know that every so often schools are recruiting you. Are you going to leave?" I answered, "No, Mom and I will be here until hell freezes over." Two months later I called her to ask, "Have you heard the weather report? It's snowing in hell." I decided that if I'm going to be intellectually honest with myself that I am attracted by where there are ideas that I find intriguing, I would have to go. I would have to leave Northwestern and move to UCI. It was hard leaving Northwestern because, again, I made so many close friends, and have such close ties. But I'm glad we moved; it's been very exciting.

You have 13 mathematical children in the U.S. and several de facto in France. What do you think makes a successful graduate advisor?

Encouraging, giving confidence to the student, and letting the student develop what he or she thinks is important. I think that a way to crush a graduate student and his or her career is to dictate the problem and outline what has to be done. This is because, really, a main role of a graduate advisor is to help students build self-confidence. The advisor should help them understand what are the serious and important issues of the field, help them learn how to select problems, and then give them encouragement, and let them develop. With different students, I have different ways of working. With some, I talk with them only occasionally to find out what they are doing. But there are others who I make come in at 9 o'clock in the morning to tell me what they did the night before, then again at 5 o'clock in the afternoon to tell me what they've done since 9 o'clock that morning! With different students, different styles: whatever it takes to help the individual student develop. I really dislike the idea of giving a problem because if you don't learn how to find a problem, when you graduate you'll find it very difficult to write that next paper beyond your thesis. There are so many people that just write up their thesis and are not sure what to do next. A very important part of the apprenticeship is to help the student learn how to find a problem, ask good questions, and not become discouraged when it takes several tries to get any results.

How do you help them mature as teachers?

I think that good mathematical research is mathematics that influences the way people in a particular research area think. And good teaching has very little to do with good penmanship at the blackboard, it's whatever it takes to influence the way students think. Also, I'm a very strong believer on the importance of exposition. After all, what we do in mathematics is very important, but most people don't know that. The key to exposition is to influence the way people think. Aha, notice how that same phrase keeps coming up? While I was at Northwestern I created a course for the first-year graduate students on how to teach. Again, the course was pretty much self-discovery. Each student would give a lecture and the other students would critique it. Then we would talk in terms of what was effective and what was not, but the key theme at all times was that your job in the classroom is to influence the way students think about the subject material. Doing mathematics, teaching, exposition, they all should be fun. If you look at it in terms of influencing the way people think about issues, then it is fun.

Is that what inspires you in your Editorship of the Bulletin, *as well?*

What is the purpose of the *Bulletin*? It's not to publish articles for a small select community. The *Bulletin* reaches 27–28,000 people. If you're writing a paper in number theory, or dynamical systems, or some other area, and if that article is written in a manner that only experts in that area can read, we're doing a disservice to more than 27,000 readers. For the *Bulletin* to play the role that the AMS believes it should play in terms of exposition, most readers should be able to read most articles, at least partway through. I must confess that we have not reached that point yet, but we've been working very hard. Again, the theme is that the *Bulletin* articles should influence how the general mathematical community thinks about mathematics.

It seems that you've had a little mathematical influence in your home as your daughters both have mathematical abilities.

Oh yes, both my daughters do. Anneli works in mathematical instruction with the emotionally, behaviorally, and mentally challenged; she teaches them mathematics all the way from elementary notions through calculus. Right now she's in the public schools. In a hospital she worked in, she was telling me about some of the abilities of the emotionally challenged. They may be emotionally troubled, but that doesn't mean they're not smart. She was teaching one of the boys, who was in 8th or 9th grade, calculus, and he was really worrying about the proofs! She and I wrote an article about the mathematics of psychology a few years back; the exchange was delightful.

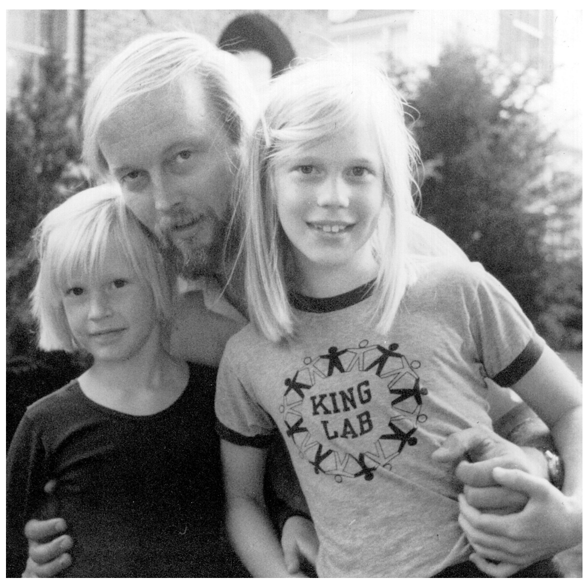

Figure 3. Saari with daughters Anneli and Katri in the 1970s outside their Evanston home.

When my older daughter Katri was in graduate school, she found some data about the propositions for the state of California, did the statistical analysis, and found that something like millions upon millions of voters were voting on about 20 propositions, yet *nobody* marked their ballot in a manner that agreed with the election results. She conjectured that that's not unusual even with a wide choice of probabilities. She had some ideas of how to prove it, and I thought it should be done a different way. So we started talking back and forth, and we proved a couple of theorems. Much fun! Then we did a second paper about semi-values. It all started when she asked me what I thought about them. My answer was not much because I didn't even know what they were. When she started explaining them to me, we found some anomalies, some paradoxes that could occur but these conclusions were isolated. We thought we should be able to prove a theorem that classifies, catches all possible paradoxes that ever could occur with semi-values of a certain class, or power indices. To explain these terms, power indices are used when you are trying to measure political power. For example, in the Senate when you have 51 Republicans and 49 Democrats, you may think they have essentially the same power. But that's not the case because Republicans can be dictatorial; they have almost complete power in the Senate because, with the voting rules, they can always win. Or another case would be in the Electoral College: how much power does the state of Wyoming have over California in the Electoral College when they're trying to elect a president? The issue is, how often can a group make the crucial difference? But there are all sorts of different power indices: the Shapley value, the Banzhof value, and on and on and on, and they can give different answers. So we started looking at the issue to try to find a general theorem that would compare all of these different indices to find everything that could ever possibly go wrong with them. We succeeded, and we had a lot of fun doing that. Working with her was educational for me. What I also liked was that this co-author would say, "I don't think you understand what you're talking about!" This co-author could be tough!

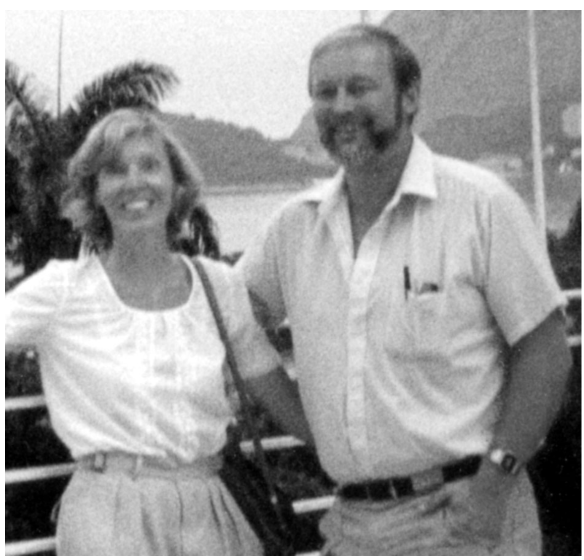

Figure 4. Saari with his wife Lillian in Brazil in the 1980s.

What does your wife think of your mathematics?

Lillian has played an important role in my career; she's been incredibly supportive. Mathematics is a blend between creativity and high-powered muscle power, technical strength. There is no way that you're going to be able to describe adequately the technical aspects to a non-mathematician. But creativity involves concepts, and concepts translate across different disciplines. Consequently, it should be possible with most ideas to describe them to someone who's not in the field. Okay, to do so, you have to learn how to tell a good, maybe wild, story that captures the basic idea in everyday terms. The process of learning how to describe an idea in this manner is not easy, but doing so has important advantages because it forces you to re-think through the concepts, to separate the concepts from the technical aspects. In that respect, Lil is my favorite and valued co-author. We talk about all the different type things that each of us does. For instance, in her work when she was still a teacher, we would talk about the problems she had. She is a very important behind-the-scenes co-author.

Lil's very patient. Of course, occasionally I get that glazed look all of a sudden, and she says "Oh, cripe, here we go again." Or when I promise I'm going to be home by 5:00, and 7:30 comes and goes and the dinner is cold, and she calls down to the office to ask "Are you still at work?" She knows precisely that that is where I am.

What have been some moments that have stood out for you in your career so far?

What are some of the highlights? Oh, after you prove a theorem that you've been working on for two or three years. Let's face it. I don't know if it's a high or it's finally "ahhhh," exhaustion! But it's really delightful. Some moments that stand out are when graduate students make a nice discovery. You can see a graduate student grow into a mathematician, from a student into a professional. Those are really delightful experiences. Working with other students, with colleagues, with other people in the field, it's fun watching people do well. I enjoyed working with the general public in trying to get them excited about the general importance and what is mathematics.

I've been interviewed a couple of times on television and radio for stories about mathematics, given lectures to CEOs about why mathematics is important to them, talked to fourth graders or high school classes, college, public lectures (where of course I start off, "Let X be a non-separable Banach space . . . !"), Rotary clubs, etc.

It's very important for the general community to appreciate the power and the importance of mathematics. I think that most people understand the power somewhat, but if they have a certain awe and view mathematics as something that is just abstract, they

Figure 5. Saari lecturing in Japan.

begin to view it as something like philosophy. While I enjoy philosophy, I most surely don't want mathematicians equated with philosophers—particularly at salary and budget time! Mathematics plays a crucial role in the driving of science, the physical sciences, the social sciences, engineering, etc. But most people are completely unaware of that, including people in the sciences. The central role of mathematics has to be known to our legislators both within the state and the US, to our administrators at our local universities, to the general public, which supports a large number of these things. We may never reach the point where there is a total understanding and acceptance, but the more inroads we can make, the better. In France they occasionally have articles at the layperson level in the newspapers that are written by prominent mathematicians. I suspect that for the most part the French population has a better understanding of what mathematics is and the importance of it. We should do more of that.

I do enjoy when I'm asked by someone what mathematics is good for. I make them tell me what they find particularly important, and ask "Where did this come from?" Then we start refining the underlining structure or ideas and very quickly you can narrow it down to where mathematics is of crucial importance, or where mathematics can make central contributions. I'm a strong believer that interesting mathematics can be found at the heart of almost anything.

A Perplexing Polynomial Puzzle

Ask an opponent to choose a polynomial $p(x)$ with nonnegative integer coefficients (of any degree). Tell them you can determine what it is with just two values: you choose a and ask for $p(a)$, then choose b and ask for $p(b)$. What is a winning strategy?

Solution on page 159. (Submitted by I. B. Keene)