

ℓ -DIVISION FIELDS AND THE CONDUCTOR OF SEMISTABLE ABELIAN VARIETIES

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ABSTRACT

This talk will be on joint work with Kenneth Kramer. I shall report on recent progress on criteria for the existence of semistable abelian varieties defined over \mathbb{Q} , with particular emphasis on odd conductor and dimension two. We obtain necessary conditions sufficient to get lower bounds compatible with Mestre's conditional bounds in dimension at most three. This is done by studying carefully the group scheme structure of the group of ℓ^n -torsion points and the ramification of the field they generate. Some questions suggested by our study will be mentioned.

Our results should be useful in testing the Langlands philosophy for surfaces over \mathbb{Q} .

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