GALOIS MODULE STRUCTURE OF GALOIS COHOMOLOGY

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Abstract. Let E/F be an extension of fields with group $\mathbb{Z}/p\mathbb{Z}$, and assume that $\xi_p \in E$. In this talk I'll discuss results and applications of the $\mathbb{F}_p[\mathrm{Gal}(E/F)]$-module structure of the groups $H^i(G_E, \mathbb{F}_p)$.

For an extension of fields E/F with $\mathrm{Gal}(E/F) = \mathbb{Z}/p\mathbb{Z}$ and $\xi_p \in F$, the $\mathbb{F}_p[\mathrm{Gal}(E/F)]$-module $E^\times/E^{\times p}$ is studied in [MS]. The tools used are elementary—Hilbert 90 and Kummer theory—and show that the module is highly stratified: with one possible exception, all indecomposable summands are either trivial or free. Using results of Waterhouse, this module decomposition can be used to prove certain automatic realization results.

The story becomes more exciting when one considers the cohomological interpretation of $E^\times/E^{\times p}$ as $H^1(G_E, \mathbb{F}_p)$. The Bloch-Kato conjecture provides tools in higher cohomology similar to those used in the study of $H^1(G_E, \mathbb{F}_p)$ and hence allows for a generalization of the results from [MS] to $H^i(G_E, \mathbb{F}_p)$ in [LMS], $i \geq 1$. The hope is that these modules can be used to investigate the possible Galois groups of maximal p-extensions $F(p)/F$. For instance, [LLMS] uses the Galois module structure of Galois cohomology of index p subgroups of $\mathrm{Gal}(F(p)/F)$ to detect whether $\mathrm{Gal}(F(p)/F)$ is a Demuškin group. A new characterization of Demuškin groups when $p = 2$ is developed there as well, and the Elementary Type Conjecture extends this characterization to all primes.

References

[LLMS] J. Labute, N. Lemire, J. Mináč, and J. Swallow. Demuskin Groups, Galois Modules, and the Elementary Type Conjecture. To appear in *J. of Algebra.* Available at *ArXiv:math.NT/0505543*

[LMS] N. Lemire, J. Mináč, and J. Swallow. Galois module structure of Galois cohomology and partial Euler-Poincare characteristics. *ArXiv:math.NT/0409484*

[MS] J. Mináč and J. Swallow. Galois module structure of pth-power classes of extensions of degree p. *Israel J. Math.* **138** (2003), 29–42.