Profinite Algebraic Geometry Seminar

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Galois Representations with Prescribed Ramification

Deligne proved that a cuspidal modular form f which is an eigenform for the Hecke algebra gives rise to an irreducible Galois representation $\rho_f\colon \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}_2(\overline{\mathbf{Q}}_\ell)$. The restriction of this representation to the inertia groups $I_p \subset \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$ for primes p encodes information about the "bad reduction" of f. In particular, the level, weight, and character are determined by these data, which one might call the "bad reduction type" of f.

If a "bad reduction type" as above is given, can one find a cusp form belonging to it? Modulo a certain parity condition, the answer is *yes*, with finitely many exceptions up to twisting by Dirichlet characters. We give a formula to count such cusp forms, which generalizes classical formulas for the dimension of spaces of cusp forms $S_k(\Gamma_0(N), \chi)$, and which also extends to the case of Hilbert modular forms. Our attack combines an "inertial Langlands correspondence" with an appropriate generalization of the Riemann-Roch formula to Galois covers of curves.