l-adic points on Modular Towers

Michel Emsalem

Université de Lille 1

In [Fr3] Mike Fried proposed a conjecture on the l-adic points in the modular tower. According to this "Harbater patching converse", projective systems of l-adic points will lie in some special components called g-p'-components.

The result we would like to discuss states that if one replaces the (coarse) Hurwitz modular tower by the Hurwitz modular stack tower, the conclusion is true (and even a little bit stronger).

In the opposite direction, using Harbater patching method we can construct projective systems of points on the Hurwitz modular tower defined over the maximal unramified extension $\mathbb{Q}_l^{ur}$ of $\mathbb{Q}_l$ and lying on coherent systems of g-p'-components. The question whether the same is possible on other type of components - an affirmative answer to this question is more or less equivalent to the negation of the "Harbater patching converse" - is still open to my knowledge. In this direction we'll give an example of points defined over $\mathbb{Q}_l^{ur}$ on non g-p'-components of arbitrary high level in the modular tower relative to the universal Frattini cover of the dihedral group D_p. But they don't form a coherent system of points.

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emsalem@math.univ-lille1.fr

Univ. Lille 1, Laboratoire Paul Painlevé, 59655 Villeneuve d'Ascq Cedex, France.