Lines in the Curriculum: A persistent obstruction to Achievement connecting Algebra and Geometry

11 AM, March 29, 2010: Conference Room at MIND Institute

Mike Fried

Emeritus UC Irvine

March 29, 2010

Summary

Today's Problem: : Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

1. **Part I**: Description of Lines in the K-14 Curriculum

Summary

Today's Problem: : Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

1. **Part I**: Description of Lines in the K-14 Curriculum
2. - I.A: There are many lines in the curriculum
   - I.B: The Point of Lines: Direction and Orientation
   - I.C: What is a line?: A Persistent Curricular Problem

Summary

Today's Problem: : Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

1. **Part I**: Description of Lines in the K-14 Curriculum
2. - I.A: There are many lines in the curriculum
   - I.B: The Point of Lines: Direction and Orientation
   - I.C: What is a line?: A Persistent Curricular Problem
3. **Part II**: California Framework Awareness?

Summary

Today's Problem: : Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

1. **Part I**: Description of Lines in the K-14 Curriculum
2. - I.A: There are many lines in the curriculum
   - I.B: The Point of Lines: Direction and Orientation
   - I.C: What is a line?: A Persistent Curricular Problem
3. **Part II**: California Framework Awareness?
4. - II.A. Where the Framework stood at the end of the '90s
   - II.B. What are lines worth?
   - II.C. The Howard Thompson Story
   - II.D. Standards and Curricular Goals

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.
2. In 4th grade? Answer: The result of a steady hand in pulling a pencil across a ruler.

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.
2. In 4th grade? Answer: The result of a steady hand in pulling a pencil across a ruler.
3. In 8th-9th grade? Answer: An expression like $y = 2x + 3$.

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.
2. In 4th grade? Answer: The result of a steady hand in pulling a pencil across a ruler.
3. In 8th-9th grade? Answer: An expression like $y = 2x + 3$.
4. In 10th grade? Answer: It is undefined, except by properties: Two points determine a unique line.

Rubric: What is a line in —?
[In practice the material is usually taught earlier than given here.]

1. In Kindergarten? Answer: A "straight" cut by scissors in the hands of a teacher or coordinated child.
2. In 4th grade? Answer: The result of a steady hand in pulling a pencil across a ruler.
3. In 8th-9th grade? Answer: An expression like $y = 2x + 3$.
4. In 10th grade? Answer: It is undefined, except by properties: Two points determine a unique line.
5. In 11th-12th grade? Answer: An expression like $y = mx + b$.

And it both continues and gets used in other courses

1. In High School Chemistry? Example: A boundary between two phases of a substance in a plot of temperature versus pressue.

And it both continues and gets used in other courses

1. In High School Chemistry? Example: A boundary between two phases of a substance in a plot of temperature versus pressue.
2. In 1st year Calculus/physics? Answer: $y = mx + b$:
   $m = \frac{d}{dx}(f(x))|_{x=x_0}$ for $f(x)$ at a point x_0.
   Approximating 4th grade line to the graph of f near x_0.

And it both continues and gets used in other courses

1. In High School Chemistry? Example: A boundary between two phases of a substance in a plot of temperature versus pressue.
2. In 1st year Calculus/physics? Answer: $y = mx + b$:
   $m = \frac{d}{dx}(f(x))|_{x=x_0}$ for $f(x)$ at a point x_0.
   Approximating 4th grade line to the graph of f near x_0.
3. In 2nd year Calculus or in physics? Answer:

$$\{(x_0, y_0, z_0) + t(u, v, w) \mid t \in \mathbb{R}\}.$$

A *parametric line*: t is a *parameter*.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

- Walk in the (u_1, v_1, w_1) direction for t_1 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_1$ feet/minute.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

- Walk in the (u_1, v_1, w_1) direction for t_1 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_1$ feet/minute.
- Then, walk in the (u_2, v_2, w_2)direction for t_2 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_2$ feet/minute.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

- Walk in the (u_1, v_1, w_1) direction for t_1 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_1$ feet/minute.
- Then, walk in the (u_2, v_2, w_2)direction for t_2 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_2$ feet/minute.
- How would I explain this to you? Answer: I might use my left arm to point. Maybe you are on an airplane (or a proton) and I'm directing you to a target up in the air.

I.B: The Point of Lines: Direction and Orientation

Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

- Walk in the (u_1, v_1, w_1) direction for t_1 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_1$ feet/minute.
- Then, walk in the (u_2, v_2, w_2)direction for t_2 minutes at the rate $\sqrt{U_1^2 + v_1^2 + w_1^2} = r_2$ feet/minute.
- How would I explain this to you? Answer: I might use my left arm to point. Maybe you are on an airplane (or a proton) and I'm directing you to a target up in the air.
- What picture might I have?: Answer: At time $t = 0$ you are at (x_0, y_0, z_0). You end up in one minute at $(x_0 + u, y_0 + v, z_0 + z)$, in 2 minutes at $(x_0 + 2u, y_0 + 2v, z_0 + 2z)$, 3 minutes, etc.

If $y = 2x + 3$ is a line in (x, y)-space,
then is $z = 2x + 3y + 4$ a line in (x, y, z)-space?

1. The first class I asked this had 57 students.

If $y = 2x + 3$ is a line in (x, y)-space,
then is $z = 2x + 3y + 4$ a line in (x, y, z)-space?

1. The first class I asked this had 57 students.
2.
   - All 57 students answered "Yes!"
   - What was their reasoning?: Answer: It looks like the formula for a line, and it has the right number of variables.

If $y = 2x + 3$ is a line in (x, y)-space,
then is $z = 2x + 3y + 4$ a line in (x, y, z)-space?

1. The first class I asked this had 57 students.
2. - All 57 students answered "Yes!"
   - What was their reasoning?: Answer: It looks like the formula for a line, and it has the right number of variables.
3. I then asked if there was a reason it might not be a line.

I.C: What is a line?: A Persistent Curricular Problem

If $y = 2x + 3$ is a line in (x, y)-space,
then is $z = 2x + 3y + 4$ a line in (x, y, z)-space?

1. The first class I asked this had 57 students.
2. - All 57 students answered "Yes!"
   - What was their reasoning?: Answer: It looks like the formula for a line, and it has the right number of variables.
3. I then asked if there was a reason it might not be a line.
4. - From Euclid: If two points are in a plane, the line *determined by them* has all its points (lies) on the plane.
   - 2nd tack: Find points on $S = \{(x, y, z) \mid z = 2x + 3y + 4\}$. What is the meaning of taking $x = 0$, then taking $y = 0$?

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

Same with $y = 0$. Do the points so confined look like lines?

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

   Same with $y = 0$. Do the points so confined look like lines?
2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at a point on S.

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

   Same with $y = 0$. Do the points so confined look like lines?
2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at a point on S.
3. Does x, y and z appearing to first power give S one property of a plane? Example: The points $(1, 1, 9)$ and $(-1, -1, -1)$ are on S. Then, all points on the line

$$L((1, 1, 9) : (-1, -1, -1))$$

   they determine are also on S. How would you check? Hint: How would you write expressions for those points?

Using the principle: Two points determine a line.

1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

   Same with $y = 0$. Do the points so confined look like lines?
2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at a point on S.
3. Does x, y and z appearing to first power give S one property of a plane? Example: The points $(1,1,9)$ and $(-1,-1,-1)$ are on S. Then, all points on the line

$$L((1,1,9) : (-1,-1,-1))$$

   they determine are also on S. How would you check? Hint: How would you write expressions for those points?
4. Euclid had no definition for a *plane*: No joke!

Using the principle: Two points determine a line.

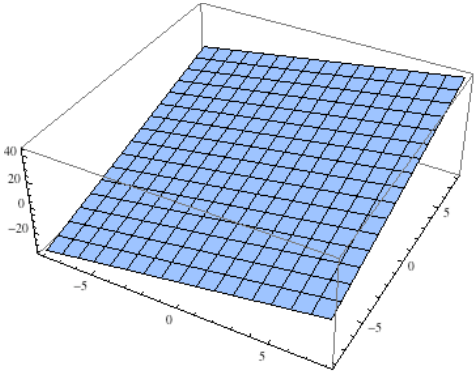
1. Setting $x = 0$ confines points *on* S to a coordinate plane.

$$S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.$$

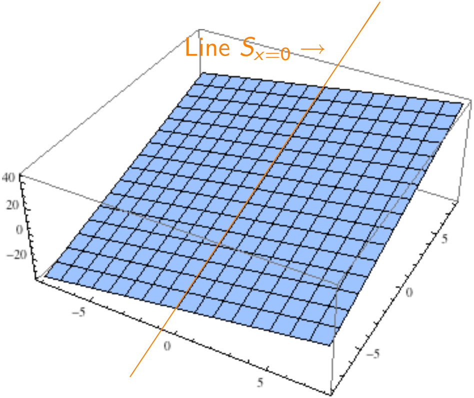
   Same with $y = 0$. Do the points so confined look like lines?
2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on S meet at a point on S.
3. Does x, y and z appearing to first power give S one property of a plane? Example: The points $(1, 1, 9)$ and $(-1, -1, -1)$ are on S. Then, all points on the line

$$L((1, 1, 9) : (-1, -1, -1))$$

   they determine are also on S. How would you check? Hint: How would you write expressions for those points?
4. Euclid had no definition for a *plane*: No joke!
5. How students perform: With similar questions, students bring out slopes. Such training won't support 3-D thinking.



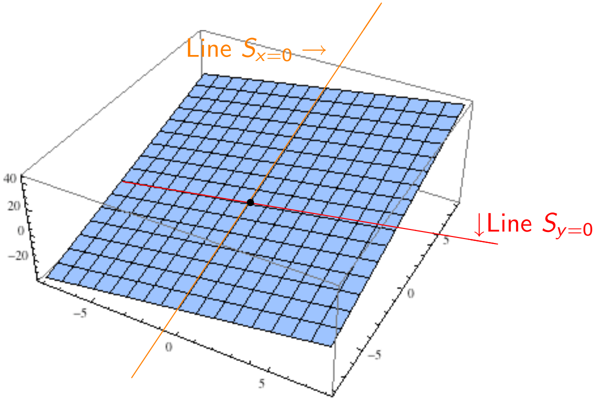
40
20
0
−20
−5
0
5
5
0
−5



Line $S_{x=0}$ →
40
20
0
−20
−5
0
5
5
0
−5

Mathematica produces S

Two lines on a Plane



In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students ... investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."

In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students . . . investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."
- p. 120: In listing of the NCTM standards for grades 5-8, for Standard 9, Algebra: "Develop confidence in solving linear equations, using concrete, informal and formal methods." There was no mention of lines in Standard 12, Geometry.

In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students . . . investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."
- p. 120: In listing of the NCTM standards for grades 5-8, for Standard 9, Algebra: "Develop confidence in solving linear equations, using concrete, informal and formal methods." There was no mention of lines in Standard 12, Geometry.
- p. 123: Unifying Ideas for Middle Grades: "Central concept is ... proportion. ... proportionality, slope, linear functions, ... the ration of *rise* over *run* is the slope of the line."

In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students . . . investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."
- p. 120: In listing of the NCTM standards for grades 5-8, for Standard 9, Algebra: "Develop confidence in solving linear equations, using concrete, informal and formal methods." There was no mention of lines in Standard 12, Geometry.
- p. 123: Unifying Ideas for Middle Grades: "Central concept is ... proportion. ... proportionality, slope, linear functions, ... the ration of *rise* over *run* is the slope of the line."
- p. 126: ... *maps* in middle school involved plane *geometry*. "in their work with maps, students explore . . . *paths* and how to specify straight paths by . . . [using] . . . distance."

Assumption in High School: Lines were mastered in Junior High

- pp. 134–174 gives the contents of the High School Curriculum. Almost no mention of lines.

Assumption in High School: Lines were mastered in Junior High

- pp. 134–174 gives the contents of the High School Curriculum. Almost no mention of lines.
- p. 143: "Students need to see the analogy between familiar frames of reference used to locate places (street patterns, building interiors . . .) and . . . coordinate geometry."

Assumption in High School: Lines were mastered in Junior High

- pp. 134–174 gives the contents of the High School Curriculum. Almost no mention of lines.
- p. 143: "Students need to see the analogy between familiar frames of reference used to locate places (street patterns, building interiors . . .) and . . . coordinate geometry."
- Yet, it was tessellations, packing problems and fractals that were brought up as relevant topics. The text didn't mention lines, and the difficulties in representing them in 3-space.

II.B. What are lines worth?

Strongest statement relating HS courses, pps 155–157: 3 headings
Connections between functions and algebra
Connections between functions and geometry
Connections between algebra and geometry
Lines were never mentioned as a persistent difficulty.

- 2nd Year Calculus [seems to stand] all alone.

Strongest statement relating HS courses, pps 155–157: 3 headings
Connections between functions and algebra
Connections between functions and geometry
Connections between algebra and geometry
Lines were never mentioned as a persistent difficulty.

- 2nd Year Calculus [seems to stand] all alone.
- Result: Almost *all* students fail: Fail to see how mathematics and engineering, physics, chemistry, economics, social sciences – that use line as a tool – relate.

II.B. What are lines worth?

Strongest statement relating HS courses, pps 155–157: 3 headings
Connections between functions and algebra
Connections between functions and geometry
Connections between algebra and geometry
Lines were never mentioned as a persistent difficulty.

- 2nd Year Calculus [seems to stand] all alone.
- Result: Almost *all* students fail: Fail to see how mathematics and engineering, physics, chemistry, economics, social sciences – that use line as a tool – relate.
- Another day's Lesson: Parametric lines give meaning to solving equations. Even if you don't have a method to solve particular equations, there still is a well-defined meaning to solving them.

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

- Administrators balked at my insisting sophomore courses impeded progress for so many students. The larger cohort of freshman calculus seemed like more "bang for the buck."

II.C. The Howard Thompson Story

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

- Administrators balked at my insisting sophomore courses impeded progress for so many students. The larger cohort of freshman calculus seemed like more "bang for the buck."
- Until I brought up Howard Thompson, they didn't realize I was talking about a total bottleneck for minority students ever participating in Mathematics, Science or Engineering.

II.C. The Howard Thompson Story

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

- Administrators balked at my insisting sophomore courses impeded progress for so many students. The larger cohort of freshman calculus seemed like more "bang for the buck."
- Until I brought up Howard Thompson, they didn't realize I was talking about a total bottleneck for minority students ever participating in Mathematics, Science or Engineering.
- Sloan Foundation Grant toward an acknowledged problem: *Nearly* 100% minority student wipeout from 1st quarter sophomore Calculus.

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

- Administrators balked at my insisting sophomore courses impeded progress for so many students. The larger cohort of freshman calculus seemed like more "bang for the buck."
- Until I brought up Howard Thompson, they didn't realize I was talking about a total bottleneck for minority students ever participating in Mathematics, Science or Engineering.
- Sloan Foundation Grant toward an acknowledged problem: *Nearly* 100% minority student wipeout from 1st quarter sophomore Calculus.
- My statistic: 51 "black" [our word then] students took the 1st quarter vector calc. in a 10 year period; one got through.

An enlightened administrator

- I told this to Dennis Galligani, a system-wide UC administrator. He asked, "Well, who was the student who got through?"

An enlightened administrator

- I told this to Dennis Galligani, a system-wide UC administrator. He asked, "Well, who was the student who got through?"
- My answer, "Howard Thompson." Dennis repeated that, "Howard Thompson? He was the only black student who got through the first quarter of vector calculus?"

An enlightened administrator

- I told this to Dennis Galligani, a system-wide UC administrator. He asked, "Well, who was the student who got through?"
- My answer, "Howard Thompson." Dennis repeated that, "Howard Thompson? He was the only black student who got through the first quarter of vector calculus?"
- http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.pdf Interactive E-Mail Assessment, MAA Vol. on Assessment, B. Gold, S.Z. Keith, and W.A. Marion, eds., Assessment in Undergraduate Mathematics, MAA Notes #49, Wash. DC, 1999, 80–84.

- "No Child Left Behind" measures the number of students who are proficient at each grade level. It requires states to adopt "challenging academic standards" in reading and math – see Title 1 – but leaves it to states to define "challenging."

- "No Child Left Behind" measures the number of students who are proficient at each grade level. It requires states to adopt "challenging academic standards" in reading and math – see Title 1 – but leaves it to states to define "challenging."
- Analogy: Let teachers repeat the text book in class, but leave developing *analyzing skills* to students on their own.

II.D. Standards and Curricular Goals

- "No Child Left Behind" measures the number of students who are proficient at each grade level. It requires states to adopt "challenging academic standards" in reading and math – see Title 1 – but leaves it to states to define "challenging."
- Analogy: Let teachers repeat the text book in class, but leave developing *analyzing skills* to students on their own.
- Result: States set their standards at widely varied levels. Obama would measure each student's academic growth, regardless of their starting performance level.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.
- What Blue-Ribbons Want: Start from scratch, get better teachers, do first years perfectly. Then, everything will be fine.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.
- What Blue-Ribbons Want: Start from scratch, get better teachers, do first years perfectly. Then, everything will be fine.
- Analogy: The Greeks were brilliant and so everything marched forward from that time.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.
- What Blue-Ribbons Want: Start from scratch, get better teachers, do first years perfectly. Then, everything will be fine.
- Analogy: The Greeks were brilliant and so everything marched forward from that time.
- Action: Common-standards effort has produced a draft.

Action Suggestions: Obama versus Many blue-ribbon committees

- Obama/Duncan: Recommend Congress overhaul NCLB. Require states to adopt "college- and career-ready standards" in reading and mathematics.
- Curricular areas must identify persistent difficulties.
- What Blue-Ribbons Want: Start from scratch, get better teachers, do first years perfectly. Then, everything will be fine.
- Analogy: The Greeks were brilliant and so everything marched forward from that time.
- Action: Common-standards effort has produced a draft.
- Assessment is everything: That requires replacing NCLB's much-criticized school rating system, known as *adequate yearly progress*, with a new accountability system.