HOD as a least branch HOD-mouse

(In models of AD^+, below IS for mice with superstrongs)

Refs (1) Normalizing IT and comparing IS (Steel's webpage)

(2) Local HOD computation

(3) LSA from l.b. Hod pairs.

(1) Premice (L[E]-pm)

- Jensen indexing
- $\lambda_E = i_E(\kappa_E)$ $\kappa_E = \mathrm{crit}(E)$
- Index of $E = \lambda^{+\mathrm{Ult}(M,E)}$
- To each premouse M we associate $k = k(M) \in \omega + 1$, which gives the degree of soundness
  $\rho(M) \overset{\mathrm{def}}{=} \rho_{k(M)+1}(M)$ $p(M) \overset{\mathrm{def}}{=} p_{k(M)+1}(M)$
  M is $k(M)$-sound.
- $\pi : M \longrightarrow N$ is elementary iff π is weakly $\Sigma_{k(M)}$-elementary
- $o(M) = \mathrm{On} \cap M$ $\omega \cdot \hat{o}(M) = o(M)$
- $l(M) = \hat{o}(M), k(M)$

$M(r,k)$ = the $N \trianglelefteq M$ s.t. $\ell(N) = \langle r, k \rangle$

<u>REM</u> $M | \langle r, 0 \rangle = M | r$ keeps $\dot{F}^{M|r} \neq \phi$

<u>Background construction $\mathbb{C}$</u>

$M^{\mathbb{C}}_{r,k}$, $\Omega_{r,k}$ by induction on $\langle r, k \rangle$.

- $M_{1,0} = (V_\omega, \in, \dots)$

  $M_{r,k+1} = \text{core}(M^{\mathbb{C}}_{r,k})$ (Need to show: the standard parameter is solid + universal.)

  $M_{r+1,k} = \underbrace{\bigoplus_k \cancel{M_{r,k}}}_{\text{the ~~}\,\text{}}$ the last core

  the rud closure of

- <u>r limit</u>

  $M^{<r} = \liminf_{\alpha < r} M_{\alpha,0}$

  $M^{*}_{r,0} = \begin{cases} M^{<r} & \text{passive case}, \dot{F}^{M_{r,0}} = \phi \\ (M^{<r}, F) & F = \dot{F}^{M_{r,0}} \text{ makes this a premouse } \underline{\text{and}}\ F \text{ has a background certificate } (F^*_r)^{\mathbb{C}} \end{cases}$

$F = F_\gamma^* \cap M^{<\gamma}$, and

F is "nice": lh = strength is inaccessible

$\underbrace{\text{strength}(F^*)}_{\lambda_{F^*}} > \lambda_F$

- $\Sigma^{\mathbb{C}}_{\gamma,k}$ = the strategy for $M^{\mathbb{C}}_{\gamma,k}$ we get by lifting $\mathcal{T}$ on $M^{\mathbb{C}}_{\gamma,k}$ to a tree $\mathcal{U}$ on V using the background extenders $(F_\gamma^*)^{\mathbb{C}}$.

This needs iterability for V.

- Lpm: <u>pm language</u> has the following symbols: - $\dot{E}$: extender sequence
  - $\dot{F}$: top extender
  - some constant symbols

<u>Lpm language</u> has additionally

- $\dot{\Sigma}$ - accumulated strategy inserted
- $\dot{B}$: new branch

Let $M = M_{\nu,0}$ where $\langle \nu, 0\rangle \overset{= \ell(M)}{}$ is <u>branch active</u>

i.e. you have a $(\eta, \ell) <_{lex} (\nu, 0)$ and a normal tree $\mathcal{T}$ on $M|(\eta,\ell)$, and $\mathcal{T}$ is of limit length and via $\dot{\Sigma}^M$ and $\dot{\Sigma}^M(\mathcal{T}, \eta, \ell)$ undefined.

$\hat{o}(M) = \beta + lh(\mathcal{T})$ some β

~~$\dot{B}^M(\alpha,$~~ Choose lex least $(\eta, \ell, \mathcal{T})$

<u>Require of lpms</u>: there is a branch b of $\mathcal{T}$ s.t.

$\dot{B}^M(\gamma) \iff \gamma = \beta + \alpha$ for $\alpha \in b$

- <u>Lpm construction</u> $\mathbb{C}$ $(M^{\mathbb{C}}_{\nu,k}, \Omega^{c}_{\nu,k})$ with same background condition for existence of F^{*}_{ν}.

For $M = M_{(\nu,0)}$ as above; branch active, $(\eta,\ell,\mathcal{T})$ the critical triple, let

$$b = \left(\Omega^{c}_{\nu,0}\right)_{\langle \eta, \ell\rangle}(\mathcal{T})$$

$$\Omega^{c}_{\nu,0} = \bigoplus_{\langle \eta, \ell\rangle \leq_{lex} \langle \nu, 0\rangle} \text{IS for } M_{\nu,0}|\langle \nu, \ell\rangle$$

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$\Omega^{\mathbb{R}}_{\nu_{10}}$ is called a "complete strategy for $M_{\nu_{10}}$"

$\dot{B}^M$ codes the b above in the above way

Properties of IS

1) Strong hull condensation

Roughly: if $\pi : H \xrightarrow{\text{elem}} V$
↑
transitive

$\pi(M) = N$

Then $\mathcal{T}$ is by Σ iff $\pi(\mathcal{T})$ is by Σ

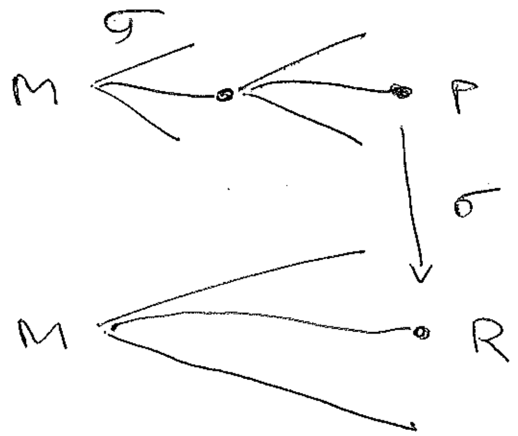
But here we drop the assumption:
$$\pi(\mathcal{T}\text{-pred}(\eta+1)) = \pi(\mathcal{T})\text{-pred}\ \pi(\eta)+1$$
(So we also need to weaken the assumption on π)

2) Normalizing well (Σ normalizes well)

Given $(\mathcal{T}, \mathcal{U})$ a stack of normal trees on M, can construct a "minimal" tree $W = W(\mathcal{T}, \mathcal{U})$ on ~~M~~ M s.t.

if $\mathcal{U}$ has last model P, we have

(in non-dropping case)



Commuting

$W(\mathcal{T},\mathcal{U})$ is the embedding normalization of $(\mathcal{T},\mathcal{U})$

REM Can construct $X = X(\mathcal{T},\mathcal{U})$ s.t. P = the last model of X; this one is called the full normalization of X.

From the construction of $W(\mathcal{T},\mathcal{U})$ it is not clear that all models of W are w.f.

- Σ normalizes well iff $W(\mathcal{T},\mathcal{U})$ exists

key points For $\mathcal{U}$ of limit length s.t. $W(\mathcal{T},\mathcal{U})$ exists, $W(\mathcal{T},\mathcal{U})$ has limit length and there is a 1-1 correspondence between cofinal branches of $W(\mathcal{T},\mathcal{U})$ and pairs (c,b) s.t.

(1) b is a cofinal branch of $\mathcal{U}$

(2) c is a maximal branch of $\mathcal{T}$ or $c = [0, \tau]_{\mathcal{T}}$ for some τ.

"Σ normalizes well" says:

$\Sigma(W(\mathcal{T}, \mathcal{U})) = a$ iff $\Sigma_{\mathcal{T}}(\mathcal{U}) = b$

<u>Def</u> V is uniquely $\vec{F}$-iterable (above κ) iff

(1) • every normal $\mathcal{T}$ on V using only extenders from $\vec{F}$ and its images has a unique cofb.

In this case we say V is uniquely iterable via $\Omega^{UBH}_{\kappa, \vec{F}}$

(2) • V has a (unique) IS ~~UBH~~ $\Omega^{UBH}_{\vec{F}}$ for "F-trees" on finite stacks of normal trees that normalizes well (and $= \Omega^{UBH}_{\kappa, \vec{F}}$ on normal $\mathcal{T}$)

<u>REM</u> If $\vec{F}$ is "coarsely coherent" then (2) follows from (1)

$i_{F_\alpha}(\vec{F}) \restriction \alpha = \vec{F} \restriction \alpha \qquad i_{F_\alpha}(\vec{F})_\alpha = \emptyset$

(Coherency condition)

Thm Suppose $\mathbb{C}$ is a pm or lpm construction with $\vec{F} = (F_\gamma^{\vee\mathbb{C}} \mid \dot{F}^{M_{\gamma,0}} \neq 0)$ being coarsely coherent. Suppose V is uniquely $\vec{F}$-iterable. then each $\Omega^{\mathbb{C}}_{\gamma,k}$ ~~has~~ normalizes well and has strong hull condensation.

$\Omega^{\mathbb{C}}_{\gamma,k}$ normalizes well: normalizing commutes with lifting to V.