2016-07-22 16:30 JOHN STEEL

Background construction $\mathbb{C}$ $(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$, $F^{*\mathbb{C}}_{\nu}$

$\mathrm{Res}_{\nu,k}[N] = \langle \eta, \ell \rangle$ for $N \trianglelefteq M_{\nu,k}$

$\sigma_{\nu,k}[N] : N \xrightarrow[e]{} M^{\mathbb{C}}_{\eta,\ell}$

$\Omega^{\mathbb{C}}_{\nu,k}$ comes from lifting V via a conversion system

$$\mathrm{lift}(\mathcal{T}, M_{\nu,k}, \mathbb{C}, \underset{\text{for } V}{\Sigma^*}) = \langle \mathcal{T}^*, \langle \eta_\gamma, \ell_\gamma \rangle_{\gamma < lh(\mathcal{T})}, \langle \pi_\gamma \mid \gamma < lh(\mathcal{T}) \rangle \rangle$$

here:

(i) $\mathcal{T}^*$ is a tree on V, models P^*_ξ, $F^*_\xi = E^*_\xi$

(ii) if $\xi <_T \nu$ and $(\xi, \nu]_T$ does not drop then

$$(\eta_\nu, \ell_\nu) = i^{\mathcal{T}^*}_{\xi,\nu}(\langle \eta_\xi, \ell_\xi \rangle)$$

$P_{\xi+1} \xrightarrow{\pi_{\xi+1}} M^{P^*_{\xi+1}}_{\eta_{\xi+1},\ell_{\xi+1}}$, $P^*_{\xi+1}$

$P_\xi \xrightarrow{F_\xi} P_{\xi+1}$, $P^*_\xi \xrightarrow{F^*_\xi} P^*_{\xi+1}$

$P_\xi \longrightarrow M^{P^*_\xi}_{\eta_\xi,\ell_\xi} = (M_{\eta_\xi,\ell_\xi})^{i^{\mathcal{T}^*}_{0,\xi}}(\mathbb{C})$

$P_\beta \longrightarrow M^{P^*_\beta}_{\eta_\beta,\ell_\beta} \in P^*_\beta$ $\mathrm{Res}_{\eta_\xi,\ell_\xi}[M^{P^*_\xi}_{\eta_\xi,\ell_\xi} \| lh\, \pi_\xi(E_\xi)] = \langle \nu, 0 \rangle$

$\Omega^{\mathbb{C}}_{\nu,k}(\mathcal{T}) = b$ iff letting $\mathrm{lift}(\mathcal{T}, M_{\nu,k}, \mathbb{C}) = (\mathcal{T}^*, -)$ then $b = \Sigma^*(\mathcal{T}^*)$.

Theorem Suppose $\mathbb{C}$ is a construction as above, (P,Σ) is an lse/lpm pair. Let $M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k}$ be defined. Suppose P iterates by Σ via a normal tree $\mathcal{T}$ to $Q \trianglelefteq M^{\mathbb{C}}_{\nu,k}$. Then $\Sigma_{\mathcal{T},M^{\mathbb{C}}_{\nu,k}} = \Omega^{\mathbb{C}}_{\nu,k}$.

Pf (Sketch) Let $\mathcal{U}$ be by both $\Sigma_{\mathcal{T},Q}$ and $\Omega^{\mathbb{C}}_{\nu,k}$ of limit length. Let

$\mathrm{lift}(\mathcal{U}, M_{\nu,k}, \mathbb{C}) = (\mathcal{U}^*, \ldots)$

let $b = \Sigma^*(\mathcal{U}^*) = \Omega^{\mathbb{C}}_{\nu,k}(\mathcal{U})$

Then $W(\mathcal{T}, \mathcal{U}^\frown b)$ is a pseudo-hull of $i^{\mathcal{U}^*}_b(\mathcal{T})$. But $i^{\mathcal{U}^*}_b(\mathcal{T})$ is a tree on P (e.g. P ctbl, so is not moved by backgrounds) by $i^{\mathcal{U}^*}_{0b}(\Sigma) = \Sigma \cap M^{\mathcal{U}^*}_b$ by universal Bairness (we make UB an assumption)

So $W(\mathcal{T}, \mathcal{U}^\frown b)$ is by Σ (since Σ has strong hull cond.) $\Rightarrow \langle \mathcal{T}, \mathcal{U}^\frown b\rangle$ is by Σ (since Σ normalises well)

To see the conclusion about pseudo-hull:

Assume Thm held for $\nu', k' \leq_{lex} \nu_0, k_0$

Had $W^*_{\nu',k'}$ normal on P, last model $M^{\mathbb{C}}_{\nu',k'}$, $\Sigma_{W^*_{\nu',k'}, M^{\mathbb{C}}_{\nu',k'}} = \Omega^{\mathbb{C}}_{\nu',k'}$ for $(\nu',k') <_{lex} (\nu_0,k_0)$

Let $\mathrm{lift}(\mathcal{U}, M_{\nu,k}, \mathbb{C}) = (\mathcal{U}^*, \langle \eta_\gamma, l_\gamma \rangle \mid \gamma < lh(\mathcal{U}), \langle \psi_\gamma \mid \gamma < lh(\mathcal{U}) \rangle)$

Let $S_\gamma = M_\gamma^{\mathcal{U}^*}$. Let $W_\gamma^* = (W^*_{\eta_\gamma, l_\gamma})^{S_\gamma}$

Construct by induction: (No dropping: $W_\gamma^* = i_{0\gamma}^{\mathcal{U}^*}(\sigma)$.)
a pseudo-hull embedding

Φ_γ from $W_\gamma = W(\sigma, \mathcal{U} \restriction \gamma+1)$
to W_γ^*

<u>Idea</u>: If $\nu \le_{\mathcal{U}} \gamma$; and

~~Λ "=" lift ($\mathcal{U} \restriction \nu+1$, M_{ν_0,k_0}, $\mathbb{C}$)~~
~~ψ "=" lift (~~

Λ = the pseudo hull embedding of W_ν into W_γ obtained from the normalization process

Λ^* = the pseudo-hull embedding on the background level of W_ν^* into W_γ^*.

Then "everything commutes", that is:

$\Lambda^* \circ \Phi_\nu = \Phi_\gamma \circ \Lambda$

"Embedding normalization commutes with the lifting process!"