(1) lpm: • Jensen indexing

• add "least missing branch" at appropriate stage; this actually means the branch for the least tree with no branch so far.

• sim type of lpm M: $\dot{E}^M$ $\dot{F}^M$ $\dot{\Sigma}^M$ $\dot{B}^M$

extender sequence / last extender / prior strategy merged / branch

• standard fine structure notation. the only difference is adding a parameter $k(M)$, the degree of soundness. M is $k(M)$-sound.

$M|\langle\nu,k\rangle$ = the $N \trianglelefteq M$ s.t. $o(N)=\nu$ and $k(N)=k$.

$M|\nu = M|\langle\nu,0\rangle$ $\quad M|\langle\nu,-1\rangle = M\|\nu$ no i.e. passive

• M-tree is a $\langle\nu,k,\mathcal{T}\rangle$ s.t. $\mathcal{T}$ is weakly normal on $M|\langle\nu,k\rangle$.

weakly normal = length increasing + non-overlapping

Dropping: need not drop to the longest initial segment.

• M-stack $S = \langle(\nu_i,k_i,\mathcal{T}_i) \mid i \le n\rangle$

• A complete strategy with scope H_θ for M is a strategy acting on all M-stacks in H_θ

• Given a strategy Ω for M and an M-stack s by Ω then $\Omega_s(t) = \Omega(s^\frown t)$

• $\Omega_{s,\langle\nu,k\rangle} = \Omega_{s^\frown\langle\nu,k,\emptyset\rangle}$

$\Omega_{s,N}$ if $N = M_\infty(s) \mid \langle r, k\rangle$

$\Omega_N = \Omega_{\phi,N}$ for $N \trianglelefteq M$

- Normalizing well, strong hull condensation } as before

Def Ω is strategy coherent iff

$$\Omega_{\langle r,-1\rangle} = \Omega_{(\langle r,0,\langle \dot{E}^{M|r}\rangle\rangle,\ \langle r,-1,\phi\rangle)}$$

Def Given $\pi : N \to N \mid \langle r,k\rangle$ and Ω a compl. I.S. for N:

$$\Omega^{(\pi,r,k)} = \pi\text{-pullback of } \Omega$$

Def Ω is self-consistent iff

(a) whenever $\langle r,k\rangle \le_{lex} \langle \eta,\ell\rangle \le_{lex} \ell(M)$

$$\Omega_{r,k} = \Omega_{\eta,\ell}^{(id,r,k)}$$

(b) the same is true for all tails Ω_s

Def (M, Ω) is a lbr hod-pair with scope H_θ iff

(1) M is a lpm

(2) Ω is a complete strategy for M with scope H_θ

(3) Ω is normalizing well IS which has strong hull condensation, is ~~self~~ strategy coherent and self-consistent

(4) For any s by Ω, $N \trianglelefteq M_\infty(s)$ we have:

$$\dot{\Sigma}^N \subseteq \Omega_{s,N}$$

we say that (N, Ω) is self-aware

REM Typically: $V \models AD^+$, M is ctbl, scope$(\Omega) = HC$

In this situation we get:

(1) Ω is pullback-consistent: if $\pi : M \xrightarrow{\mathcal{T}} N$ is an iteration map by Ω then

$$\Omega^{\mathcal{T}}_{\langle \ell(m), \mathcal{T} \rangle, N} = \Omega$$

(2) Ω is positional: For s, t by Ω s.t.

$$N \trianglelefteq M_\infty(s), \quad N \trianglelefteq M_\infty(t)$$

we have $\Omega_{s,N} = \Omega_{t,N}$

An lpm construction $\mathbb{C}$ has ~~$M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k}$~~ $M^{\mathbb{C}}_{\nu,k}$, $\Omega^{\mathbb{C}}_{\nu,k}$

(Given Σ^* for V assume V is uniquely iterable for normal trees (nice ones). Hence strategy extends to stacks.

$(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$ are lbr hod pairs.

Also let F^*_ν are the background extenders for $\dot{F}^{M_{\langle \nu, 0 \rangle}}$ if exist

$(+)_{\nu,k}$ Granted $(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$ exists:

(i) $\rho(M_{\nu,k})$ is solid

(ii) For $\rho = \rho(M_{\nu,k})$, $M_{\nu,k} | \rho^{+M_{\nu,k}} \trianglelefteq \mathrm{Hull}^{M_{\nu,k}}_{k+1}(\rho \cup p(M_{\nu,k}))$

i.e. $M_{\nu,k+1} = \mathrm{core}(M_{\nu,k})$ exists; in other words, $p(M_{\nu,k})$ is universal

$(+)_{0,-1}$ if F^*, G^* can serve as background for $(M_{\nu,-1}, F)$, $(M_{\nu,-1}, G)$ then $F = G$

$(+)_{\nu,k}$ is done by induction on ν, k

<u>Comparison lemma</u> Let δ be Woodin and assume V is uniquely normally iterable for mice trees in V_δ.
Let (P, Σ) be lbr hod pair $P \in V_\delta$ and Code(Σ) is δ-UB. $(P \in HC^{V_\delta})$. Let $\mathbb{C}$ be an lpm construction with F^*_ν's in V_δ, a "maximal one", that is, $F^*_\nu \neq 0$ whenever possible. Then there is $\langle \nu, k \rangle$ s.t.

(1) (P, Σ) iterates to $(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$

(2) (P, Σ) iterates strictly past $(M_{\nu',k'}, \Omega_{\nu',k'})$ whenever $(\nu', k') <_{lex} (\nu, k)$

<u>Dodd-Jensen Lemma</u> For lbr hod pair (M, Ω) and a stack s by Ω, $N \trianglelefteq M_\infty(s)$ and $\pi : M \to N$ s.t. $\Omega^\pi_{s,N} = \Omega$. Then M-to-N does not drop, so we have $i : M \to N$ an iteration map, and

$i(\eta) \leq \pi(\eta)$ f.a. $\eta \in M \cap On$.

(Note: the assumption on π can be briefly written as: $\pi : (M, \Omega) \to (N, \Omega_{s,N})$)

<u>Corollary</u>: If π above itself is an iteration map by Ω, say the stack it t so $N \trianglelefteq M_\infty(t)$ and $M = M_\infty(t)$ and M-to-N does not drop and $\Omega_{s,N} = \Omega_{t,N}$ then $\Omega = \Omega^\pi_{t,N}$ (by pullback consistency)
$= \Omega^\pi_{s,N}$
thus $\pi = i$.

<u>Corollary</u> ~~HPC~~ (AD+), For (P, Σ) an lbr hod pair with scope HC.

<u>Rem</u> Strategies are automatically Suslin-co-Suslin; this follows from strong hull condensation.

$M_\infty(P, \Sigma)$ = dir. lim. of all Σ-iterates of (P, Σ) results

<u>REM</u> Each $M_\infty(P, \Sigma)$ is OD. Hence $(P, \Sigma) \equiv (Q, \Gamma)$ iff $M_\infty(P, \Sigma) = M_\infty(Q, \Gamma)$

Have a common iterate

$\leq^*$ is a Pwo by D-J