2016-07-26 11:15 JOHN STEEL

Correction (a) Regarding Farmer's Question

Can E^{Ψ}_{Θ} applied to M^{Ψ}_{α} for Θ unstable.

What if you have to drop? Note: $\gamma+1$ is stable

(b) Ralf's question: Already corrected in the notes.

Theorem. Assume AD^+, let (M, Ψ) be an lbr hod pair with scope HC. Let code(Ψ) be Suslin-co-Suslin. Suppose $M \models \lambda$ is a limit of Woodins + ZFC

Let g be $Col(\omega, <\lambda)$-generic / M, $\mathbb{R}^*_g = \bigcup_{\alpha<\lambda} \mathbb{R}^{M[g \upharpoonright \alpha]}$

and $Hom^*_g = \{ p[T] \cap \mathbb{R}^*_g \mid M[g\upharpoonright\alpha] \models T$ is $<\lambda$ a.c.$\}$.

Then $HOD^{L(\mathbb{R}^*_g, Hom^*_g)}$ is an lpm

REM $L(\mathbb{R}^*_g, Hom^*_g) \models AD_{\mathbb{R}}$

This follows that the restrictions of strategies to $\mathbb{R}^{V[g\upharpoonright\alpha]}$ are Wadge cofinal in Θ. (This gives that all sets are Suslin)

REM If λ is a limit of cutpoints in M then $HOD^{L(\mathbb{R}^*_g, Hom^*_g)}$ is an iterate of $M \mid \lambda$.

Q What is the determining counterpart of such a hod mouse?

Proof We use the following:

Theorem: Assume AD^+. Let (P, Σ) be an lbr hod pair with scope HC, and ~~let~~ Σ^{rel} assume

(a) Code(Σ^{rel}), $\neg$ Code(Σ^{rel}) are κ-Suslin where $\kappa = |M_\infty(P, \Sigma)|$

(b) Code(Σ^{rel}) is not α-Suslin for any $\alpha < \kappa$

$\mathcal{T}$ is by Σ^{rel} iff $\mathcal{T}$ is by Σ and there is an $\mathcal{Y} \supseteq \mathcal{T}$ normal by Σ with last model Q s.t. P-to-Q does not drop.

A stack $\vec{\mathcal{T}}$ is by Σ^{rel} iff every $\mathcal{T}_i$ is.

(b) above follows by Kunen-Martin Thm.

Regarding (a) above: Idea: if (P, Σ) has b.c. $\mathcal{T}^\frown b$ is a normal non-dropping tree by Σ iff

$$\exists \sigma : M_b^{\mathcal{T}} \longrightarrow M_\infty(P, \Sigma)$$

s.t. letting $\pi^{\Sigma}_{P,\infty} : P \longrightarrow M_\infty(P, \Sigma)$ be the direct limit map, $\pi^{\Sigma}_{P,\infty} = \sigma \circ i_b^{\mathcal{T}}$

In the general case there is a normal tree $\mathcal{U}$ on P that is by Σ with last model $M_\infty(P, \Sigma)$. (We have a generic stack $\langle \mathcal{Y}_i \mid i < \omega \rangle$ by Σ with last model $M_\infty(P, \Sigma)$). Can fully normalise this stack. Yields a single normal $\mathcal{U}$ on P with last model $M_\infty(P, \Sigma)$. $\mathcal{U}$ is by Σ in that all its countable elementary submodels are by Σ.

Given $\mathcal{T}^\frown b$ by Σ countable, b non-dropping: Search for a "weak hull embedding" Ψ from $\mathcal{T}^\frown b$ into $\mathcal{U}$.

(*) $\mathcal{T}^\frown b$ is by Σ iff $\exists$ such Ψ

$\Leftarrow$ in (*) follows by strong hull condensation

This is a comparison argument similar to that of the UBH proof. Use Dodd-Jensen property in place of pointwise definability. ~~See~~ See "Local HOD computation".

$\Rightarrow$ In $(*)$: If it is by Σ we have

$$P \xrightarrow{\sigma} M_b^{\sigma} \longrightarrow \overset{\vartheta_i}{\cdots\cdots} \longrightarrow M_\infty(P, \Sigma)$$

Full normalization gives a single normal W on M_b^{σ} with last model $M_\infty(P, \Sigma)$

Then normalize the stack $\sigma \frown W$, denote it by $X(\mathcal{T}^b, W)$ (full normalization)

Hence there is a weak hull embedding from $\mathcal{T}^b$ into $\mathcal{U}$ (this is what full normalization gives this) $\boxtimes$

<u>Now back to HOD computation</u> Recall

$M \models \lambda$ is a limit of Woodins

$\mathbb{R}^*_g$, Hom^*_g from $D(M, <\lambda)$

<u>Lemma</u> For each $r < \lambda$, $k \in \omega$

$$\tau^g_{\langle r,k\rangle} \overset{\text{def}}{=} \tau_{\langle r,k\rangle} \cap (HC^*)^g$$

is in Hom^*_g.

<u>Proof</u> Use generic interpretability. The term gives UB representation.

Lemma These strategies are Wadge cofinal in Hom^*_g.
This is because any such set can be reduced to one of these strategies.
Let T, T^* be a.c., in $M[g]$. Use genericity iterations and corresponding images of T, T^* to decide if a given real is in our set.

Working in $L(\mathbb{R}^*_g, \mathrm{Hom}^*_g)$, say (P, Σ) an lbr hod pair is full iff (Σ is Suslin-co-Suslin) and
(a) $P \models ZFC^-$ and has a largest cardinal δ; $k(P) = 0$
(b) whenever s is a P-stack by Σ with

$M_\infty(s) = Q$, P-to-Q does not drop
and $i_s : P \to Q$ is the iteration map then
there is no lbr hod pair (R, Φ) s.t.
(Φ is Suslin-co-Suslin) and $Q \trianglelefteq^{cutpt} R$
and $\rho(R)$ $i_s(\delta)$ and $\Phi_Q \leq \Sigma_s$.

Lemma Let η be a successor cardinal of M, then $(M|\eta, \Psi^g_{M|\eta})$ is a full hod pair in $L(\mathbb{R}^*_g, \mathrm{Hom}^*_g)$
(Any c.ex to fullness is OD, as it is unique by standard argument - this uses that Q is a cutpoint.)

In $L(\mathbb{R}^*_g, \mathrm{Hom}^*_g)$ define
$\mathcal{F} = \{(P, \Sigma) \mid (P, \Sigma)$ is a full lbr hod pair$\}$

$<^*$ on $\mathcal{F}$ is defined by

$(P,\Sigma) <^* (Q,\Psi)$ iff

$\exists (R,\Phi)\in\mathcal{F}$ s.t. $(R,\Phi) \trianglelefteq^{\text{cutpt}} (Q,\Psi)$

and (P,Σ) iterates to (R,Φ) without dropping

$M_\infty = \text{dir lim } \mathcal{F}$

Lemma $M_\infty \subseteq HOD|\Theta^{L(\mathbb{R}^*_g,\, Hom^*_g)}$

$M_\infty = \bigcup\{M_\infty(P,\Sigma) \mid (P,\Sigma)\in\mathcal{F}\}$

This is clear

Lemma $HOD|\Theta \subseteq M_\infty$

Pf Standard argument. (Sketch)

Let $A\subseteq\alpha$, $\alpha<\Theta$.

$\beta\in A \iff L(\mathbb{R}^*_g, Hom^*_g)\models\varphi(\beta,\gamma_0)$.

Can find $(P_0,\Sigma_0) = (M|\eta, \Psi_{M|\eta})$ in $\mathcal{F}$

with $\pi_{P_0,\infty}(\bar\alpha)=\alpha$.

$$\begin{array}{ccccccc}
M_0 = N_0 & \longrightarrow & N_1 & \longrightarrow & N_k & \xrightarrow{j_k} & \cdots \to N_\omega\\
id\uparrow & \rightrightarrows & \uparrow & \rightrightarrows & \uparrow & \rightrightarrows \cdots & \uparrow \\
M = M_0 & & M_1 & & M_k & & \\
\trianglerighteq & & \trianglerighteq & & \trianglerighteq & & \\
P_0 & \longrightarrow & P_1 & \longrightarrow & P_k & \longrightarrow \cdots & P_\omega = M_\omega(P_0,\Sigma_0)
\end{array}$$

Each P_k is in $L(\mathbb{R}^*_g, Hom^*_g)$. Generically iterate each ~~P_k~~ M_k to absorb $\mathbb{R}^*_g$ and simultaneously copy the iteration of M_k into the one of M_{k+1}

Have $j_k : N_k \longrightarrow N_{k+1}$

$L(\mathbb{R}^*_g, \mathrm{Hom}^*_g) = L(\mathbb{R}^*_{h_i}, \mathrm{Hom}^*_{h_i})$ for h_i which is

$\mathrm{Coll}(\omega, <\lambda)$ - generic / N_i

$\exists k \ \forall l \geq k \quad j_l(\sigma_0) = \sigma_0$.

$$A^{N_l}_l = \{ \varphi < j_{0l}(\cancel{\ }\alpha) \mid \Vdash_{\mathrm{Coll}(\omega, <\lambda)} L(\mathbb{R}^*, \mathrm{Hom}^*) \models \varphi(\pi_{p_{l,\infty}}(\xi), \sigma_0) \}$$

$$\pi_{p_{k,\infty}}(A^{N_k}_k) = A$$