MATH 13 WINTER 2016 HOMEWORK 2

Due: Friday January 29 Please turn in at the lecture.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your arguments clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding.

When preparing the homeworks, please follow the **Rules for homeworks** on the course webpage under **Course information and policies** and also the guidelines under **Grading**. In particular, keep in mind the **Aspects of grading** in the **Grading** section.

1. An integer is a **square** if and only if it is of the form x^2 for some integer x. Write down the following statements in rigorous mathematical language using quantifiers and logical connectives.

(a) **(1pt)** Integer y is a square.
(b) **(1pt)** Some integers are not squares.
(c) **(1pt)** Integer y is a sum of two squares.
(d) **(1pt)** Every integer is a sum of two squares.
(e) **(1pt)** Negation of (d).

Prove or disprove:

(f) **(1pt)** (b) holds.
(g) **(2pt)** (d) holds.

Caution! When forming negation **do not** simply put the negation sign in front of the statement, but reformulate the statement according to rules for negating quantified statements; see Book, Theorem 2.28. At the same time write down this negation in a way that it has reasonable mathematical meaning.

2. Consider the parametric equation

$$3x^2 + px + 1 = 0 \tag{1}$$

Write down the following statements in rigorous mathematical language using quantifiers and logical connectives.

(a) **(1pt)** Equation (1) has an integer solution for any integer parameter p.
(b) **(1pt)** Equation (1) has an integer solution for some integer parameter p.
(c) **(1pt)** There is an integer parameter p for which equation (1) has more than one solution.
(d) **(1pt)** Negation of (c).

Prove or disprove:

(e) **(1pt)** (a) holds.
(f) **(2pt)** (c) holds.

Note: For forming a negation use the same guidelines as for Problem 1 above.

3. Consider a function f with real arguments and real values. We say that:

- f is **unbounded from above** if and only if f attains arbitrarily large values. Otherwise we say that f is **bounded from above**.
- f is **unbounded from above on the interval** (a, b) if and only if f attains arbitrarily large values while the argument ranges over the interval (a, b). Otherwise we say that f is **bounded from above on the interval** (a, b).

So for instance f is unbounded from above if and only if f is unbounded from above on the interval $(-\infty, +\infty)$.

Write down the following statements in rigorous mathematical language using quantifiers and logical connectives.

(a) **(1pt)** f is unbounded from above.
(b) **(1pt)** f is unbounded from above on the interval $(0, 1)$.
(c) **(1pt)** f is bounded from above.
(d) **(1pt)** f is bounded from above on every interval of the form $(0, a)$ where a is a positive real number.

Prove or disprove:

(e) **(1pt)** The conjunction (a)$\wedge$(d) holds.