AXIOMS OF PREDICATE CALCULUS

(TAUT) **ALL TAUTOLOGIES**

EQUALITY AXIOMS

(E1) $t = t$ for any term t

(E2) $t = t' \longrightarrow (\varphi(x/t) \longleftrightarrow \varphi(x/t'))$ φ atomic

QUANTIFIER AXIOMS

(Q) $\varphi(x/t) \longrightarrow (\exists x)\varphi$ t substitutable for x in φ

RULES OF INFERENCE

MODUS PONENS

(MP) $$\frac{\varphi \qquad \varphi \longrightarrow \psi}{\psi}$$

QUANTIFIER RULE

(QR) $$\frac{\varphi \longrightarrow \psi}{(\exists x)\varphi \longrightarrow \psi}$$ x not free in ψ

ROBINSON ARITHMETIC Q

(S1) $$\neg\, \mathbf{S}(x) = \mathbf{0}$$

(S2) $$\mathbf{S}(x) = \mathbf{S}(y) \;\longrightarrow\; x = y$$

(L1) $$x < \mathbf{S}(y) \;\longleftrightarrow\; x < y \vee x = y$$

(L2) $$\neg\, x < \mathbf{0}$$

(L3) $$x < y \;\vee\; x = y \;\vee\; y < x$$

(A1) $$x + \mathbf{0} = x$$

(A2) $$x + \mathbf{S}(y) = \mathbf{S}(x + y)$$

(M1) $$x \cdot \mathbf{0} = \mathbf{0}$$

(M2) $$x \cdot \mathbf{S}(y) = (x \cdot y) + x$$