MATH 281A FALL 2016 FINAL EXAM

Due date: Friday December 9, 2016

Rules: Write as efficiently as possible: Include all relevant points and think carefully what to write and what not. Use common sense to determine what is the appropriate amount of details for a course at this level. Quote any result from the lecture you are referring to; do not reprove the result. Each problem indicates maximum allowed length; this is usually much more than needed. If you type, do not use font smaller than 10pt.

I will not grade any text that exceeds the specified length.

1. (2/3 page) Assume $\mathcal{N}$ can be well-ordered. Prove that there is an undetermined set.

Hint. Diagonalize against all strategies.

2. (2/3 page) Assume $\mathcal{N}$ can be well-ordered. Prove that there is a set $A \subseteq \mathcal{N}$ such that:

(a) Both A and $\mathcal{N} \setminus A$ have size $2^{\aleph_0}$.
(b) Neither A nor its complement contains a perfect subset.

Hint. This is a diagonalization similar to that in Problem 1.

3. (1 page) Let Γ be a pointclass closed under recursive substitution. Prove:

$$\Gamma \text{ is determined} \iff \check{\Gamma} \text{ is determined.}$$

4. (1/2 page) Let Γ be a pointclass closed under $\vee$ and $\forall^{\mathbb{R}}$. Recall $\Delta = \Gamma \cap \check{\Gamma}$. Assume $A, B \subseteq \mathcal{N}$ are such that $A \in \Gamma \setminus \Delta$, $B \in \check{\Gamma}$ and $B \subseteq A$. Assume further that there is a Γ norm φ on A such that $\operatorname{rng}(\varphi) = \lambda \in \mathbf{On}$.

Prove that there is an ordinal $\alpha < \lambda$ such that $\varphi(x) \leq \alpha$ whenever $x \in B$.