EXERCISES 27.6.2012

(1.) Show that, given an acceptable structure $M = \langle J^B_\alpha, D\rangle$, ρ^2_M computed in the classical way is the same as ρ^2_M computed using the Σ^*-approach. Also do the same for P^2_M, R^2_M.

(2.) Write p^n_M = the $<^*$-least element of P^n_M. Let $q = p^2_M \cap [\rho^1_M, \omega\alpha)$. Is $q = p^1_M$? Is this true if we assume $P^1_M = R^1_M$?

(3.) Let $\sigma : J_\tau \to_{\Sigma_0} J_\tau$, cofinally where κ is the largest cardinal in J_τ. Assume $cf^V(\tau) > \omega$. Let τ be a cardinal in J_α. Show that $Ult(J_\alpha, \sigma)$ is well-founded.

(4.) More generally (?): In (3) drop the assumption that $\tau = \kappa^{+J_\alpha}$, just assume τ is a cardinal in J_α. Add the assumption $\rho^1_{J_\alpha} \le \tau$.

Hint for (3) + (4): Let $\langle \alpha_i, f_i\rangle_{i<\omega}$ be a descending sequence in the ultrapower. Form a countable elementary substructure X of H_θ (θ large) s.t. J_α and all f_i are in X, and apply the interpolation lemma to the inverse of the collapsing isomorphism.