EXERCISES 28.6.2012 (1)

(1.) Assume $\sigma : Q \to Q'$ is Σ_0-cofinal where $Q = H^M_\tau$ and τ is regular in M. Let γ be the least s.t. there is a partial function $f : \gamma \to \tau$ s.t.

(a) $\gamma < \tau$ and f is cofinal in τ

(b) f is $\Sigma_1^{(n)}$

Let $M' = \mathrm{Ult}_n(M, \sigma)$. Show that the ultrapower map $\tilde{\sigma} : M \to M'$ is cofinal at the level n.

(2) Prove by induction on α that each J_α is acceptable and sound.

(3) Prove for any acceptable structure M that the following two are equivalent:

(a) M is sound, i.e. $R^n_M = P^n_M$ all n

(b) $\rho^n_M \in R^n_M$ all n

(3) Prove the fine structural variant of the interpolation lemma: Assume $\sigma : M \to M'$ is $\Sigma_0^{(n)}$. Let $Q = H^M_\tau$ (*), $\tilde{Q} = \bigcup_{x \in Q} \sigma(x) = H^{M'}_{\tilde{\tau}}$ where $\tilde{\tau} = \sup \sigma[\tau]$. Let n be s.t. $\rho^n_M \geq \tau$. Then there is a structure $\tilde{M}$ and maps

$\tilde{\sigma} : M \to \tilde{M}$ $\quad \sigma' : \tilde{M} \to M$

s.t.

(i) $\tilde{\sigma}$ is $\Sigma_0^{(n)}$-preserving and σ' is $\Sigma_1^{(n-1)}$-preserving

(ii) $\tilde{\sigma} \upharpoonright Q = \sigma \upharpoonright Q$ and $\tilde{\sigma}(\tau) = \tilde{\tau}$

(iii) $\sigma' \upharpoonright \tilde{\tau} = \mathrm{id}$ and $\sigma'(\tilde{\tau}) = \tau$

(iv) $\sigma = \sigma' \circ \tilde{\sigma}$

Moreover, if $\tilde{\sigma}$ is cofinal then

(v) σ' is $\Sigma_0^{(n)}$-preserving

(*) assume τ is regular in M.