Nonreciprocal magnetic photonic crystals

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We study band dispersion relations $\omega(\vec{k})$ of a photonic crystal with at least one of the constitutive components being a magnetically ordered material. It is shown that by proper spatial arrangement of magnetic and dielectric components one can construct a magnetic photonic crystal with strong spectral asymmetry (nonreciprocity) $\omega(\vec{k}) \neq \omega(-\vec{k})$. The spectral asymmetry, in turn, results in a number of interesting phenomena, in particular, one-way transparency when the magnetic photonic crystal, being perfectly transparent for a Bloch wave of frequency Ω, ''freezes'' the radiation of the same frequency Ω propagating in the opposite direction. The frozen radiation corresponds to a Bloch wave with zero group velocity $\partial\omega(k)/\partial k = 0$ and, in addition, with $\partial^2\omega(k)/\partial k^2 = 0$.

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I. INTRODUCTION

Spatially periodic composite dielectric structures, known as photonic crystals, have been a subject of extensive research for their remarkable spectral properties. As a consequence of spatial periodicity, the frequency spectrum of a photonic crystal has the band-gap structure. In every band there is a dispersion relation $\omega(\vec{k})$ relating the wave vector $\vec{k}$ of the corresponding Bloch wave to its frequency ω. The spectral band structure of a typical photonic crystal is similar to that of electronic energy spectra in semiconductors and metals. In particular, it may have complete spectral gaps also called forbidden frequency bands [1]. The presence of spectral gaps constitutes the basis for many existing practical applications of photonic crystals. For the last decade an extensive literature has evolved on various theoretical, experimental, and practical aspects of the problem (see, for instance, [2–6] and references therein). At the outset, the researchers dealt almost exclusively with photonic crystals composed of two isotropic dielectric materials. Then, the physical nature of the constitutive components was diversified to include anisotropic media, conductors, and even superconductors. Rather recently some publications have emerged in which magnetic photonic crystals are considered. Let us take a brief look at some of them.

In [7] and [8] the authors study some two-dimensional (2D) and three-dimensional (3D) photonic crystals with one of the two constitutive components displaying, along with electric permittivity $\hat{\varepsilon}$, an appreciable magnetic permeability $\hat{\mu}$. The main focus of these investigations is the effect of magnetic permeability on the position and the width of electromagnetic band gaps.

References [9–12] are devoted to the effect of *Faraday rotation* of light polarization in dielectric composite media. This phenomenon has essentially a magnetic nature. Indeed, such an effect, being time-odd, does not exist in a nonmagnetic substance in the absence of an external magnetic field. One of the most important results of these publications is that the magnetic composite media may exhibit the effect of Faraday rotation, which is much stronger than that of the magnetic constitutive components taken separately. This is particularly important in the optical and infrared frequency range, where the magnitude θ_F of Faraday rotation in a uniform substance does not exceed 10^{-1} deg per wavelength $\lambda \approx 1\ \mu$m. According to [10], the effectiveness of such an enhancement may reach 10^2 or even 10^3.

The Faraday rotation is the most known electromagnetic phenomenon that has an essentially magnetic origin. It is prohibited by symmetry unless the substance has some sort of long-range magnetic order, or if an external magnetic field is introduced, [13,14]. There exists another important property of the electromagnetic spectrum that is also unique to magnetically ordered media. This phenomenon is referred to as *spectral nonreciprocity*

$$\omega(\vec{k}) \neq \omega(-\vec{k}). \tag{1}$$

This remarkable effect was theoretically predicted more than three decades ago in [15] (see also [16–18] and references therein). It was shown to exist in a special class of magnetically ordered crystals called *magnetoelectrics*. The fundamental property of magnetoelectric crystals is the existence of the linear magnetoelectric response

$$\hat{\chi} = \frac{\partial \vec{D}}{\partial \vec{H}}, \quad \hat{\chi}^T = \frac{\partial \vec{B}}{\partial \vec{E}}, \tag{2}$$

where $\hat{\chi}$ is a time-odd tensor which in most cases is asymmetric

$$\hat{\chi} \neq \hat{\chi}^T.$$

More information on magnetoelectric media can be found in [13,19,20,16,14,21,22]. A graphic example of the fundamental relation between the linear magnetoelectric response $\hat{\chi}$ of anisotropic media and the property (1) of the spectral nonreciprocity is presented in the Appendix. An extensive discussion on this topic can be found in [16].

The overwhelming majority of known dielectric materials do not display linear magnetoelectric effects and have reciprocal electromagnetic spectra. The simple reason for this is that if the medium supports space-inversion symmetry $\mathcal{I}$

and/or time-reversal symmetry $\mathcal{R}$, then all nine components of the tensor $\hat{\chi}$ are zeros [13,14]. Indeed, taking into account that

$$\vec{D}=-\mathcal{I}\vec{D}, \quad \vec{E}=-\mathcal{I}\vec{E}, \quad \vec{H}=\mathcal{I}\vec{H}, \quad \vec{B}=\mathcal{I}\vec{B},$$

$$\vec{D}=\mathcal{R}\vec{D}, \quad \vec{E}=\mathcal{R}\vec{E}, \quad \vec{H}=-\mathcal{R}\vec{H}, \quad \vec{B}=-\mathcal{R}\vec{B},$$

we have from the definition (2)

$$\mathcal{I}\hat{\chi}=-\hat{\chi}, \quad \mathcal{R}\hat{\chi}=-\hat{\chi}. \tag{3}$$

Therefore, the linear magnetoelectric effect (2) is incompatible with space-inversion and time-reversal symmetries [13]. All popular ferromagnets and ferrites, being magnetically ordered, do not support the time-reversal symmetry $\mathcal{R}$, but they do support space inversion $\mathcal{I}$ and, therefore, have a zero magnetoelectric response $\hat{\chi}$. On the other hand, numerous dielectric materials, which do not have the space-inversion symmetry (those comprise all ferroelectrics, piezoelectrics, and optically active media), normally support the time-reversal symmetry $\mathcal{R}$ and, for this reason, have no magnetoelectric response $\hat{\chi}$ either. Only a few ferrites and antiferromagnets have neither of the two symmetries and may support linear magnetoelectric response $\hat{\chi}$, as well as spectral nonreciprocity (1). In addition to being extremely rare, the known magnetoelectric crystals have poor quality and display a very low magnitude of linear magnetoelectric response which normally is a small fraction of a percent [16]. For this reason, their remarkable electromagnetic properties, featuring spectral nonreciprocity, have not found any significant application in optics and microwave technology. The effect appears to be too small to be of any serious practical interest.

Turning back to magnetic photonic crystals, let us consider the possibility of achieving *strong* spectral nonreciprocity in a composite dielectric media made up of nonmagnetoelectric components. As shown below, under certain conditions magnetic photonic crystals can support appreciable spectral nonreciprocity even in the absence of the magnetoelectric effect (2). Therefore, from now on we restrict our consideration to periodic structures composed only of generic readily available dielectric components (ferrites and anisotropic dielectrics) satisfying generic constitutive relations[1]

$$\vec{D}=\hat{\varepsilon}\vec{E}, \quad \vec{B}=\hat{\mu}\vec{H} \tag{4}$$

without any ''exotic'' magnetoelectric terms (2). This implies that each of the uniform constitutive components, if it fills the entire space, has a perfectly reciprocal electromagnetic spectrum

[1]Hereinafter, the symbols $\vec{D}$, $\vec{E}$, $\vec{B}$, and $\vec{H}$ denote the alternating components of the electromagnetic field. These components are associated with linear electromagnetic waves. The static components of the fields and polarizations, if they occur, will be supplied with the subscript 0. For more on this see Secs. III A and VI.

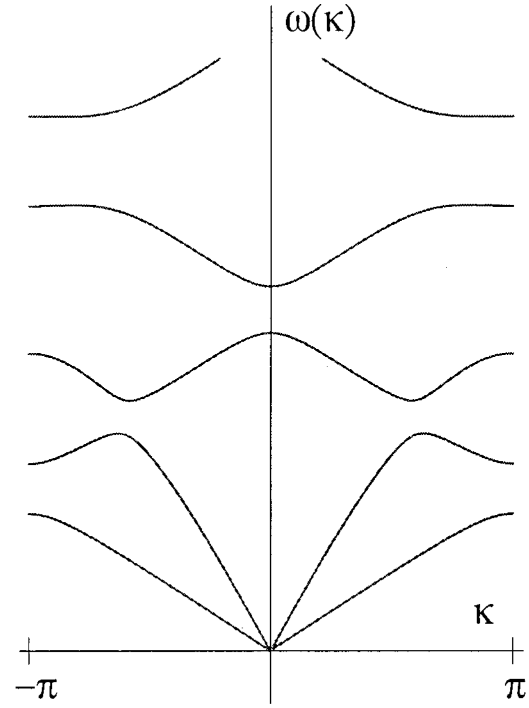

FIG. 1. Reciprocal electromagnetic spectrum of the three-layered periodic stack in Fig. 6 with $\varphi=0$. The numerical set 1 from Eq. (82); $F/A=1$.

$$\omega(\vec{k})=\omega(-\vec{k}), \tag{5}$$

while a magnetic photonic crystal composed of such reciprocal components may support an essentially nonreciprocal spectrum (1). In other words, the property (1) of spectral nonreciprocity can be achieved in magnetic photonic crystals *solely by proper space arrangement of the constitutive components.*

The first problem addressed in this paper is how to design a photonic crystal with strong spectral nonreciprocity (1) using constitutive components, each of which has a perfectly reciprocal spectrum (5). It turns out that although such a possibility is unique to magnetic photonic crystals, the spectral nonreciprocity (1) by no means occurs automatically as soon as photonic crystal includes some magnetic components. Quite the opposite, only rather special periodic arrays of magnetic and dielectric components can produce the result. For example, none of the magnetic photonic crystals considered in quoted publications meet the criteria we derived, and, hence, all of them have regular reciprocal spectra, similar to those shown in Figs. 1 and 2. In Secs. II and III we consistently develop guidelines for the proper spatial array of the constitutive components that would yield nonreciprocal spectra.

Sections IV, V, and VI are devoted to the spectral analysis of several specific examples of one-dimensional (1D) magnetic photonic crystals. Those examples graphically demonstrate important features of nonreciprocal electromagnetic spectra, as well as their dependence on the key geometric and material parameters of magnetic photonic crystals. The spectral calculations also reveal some fundamental limita-

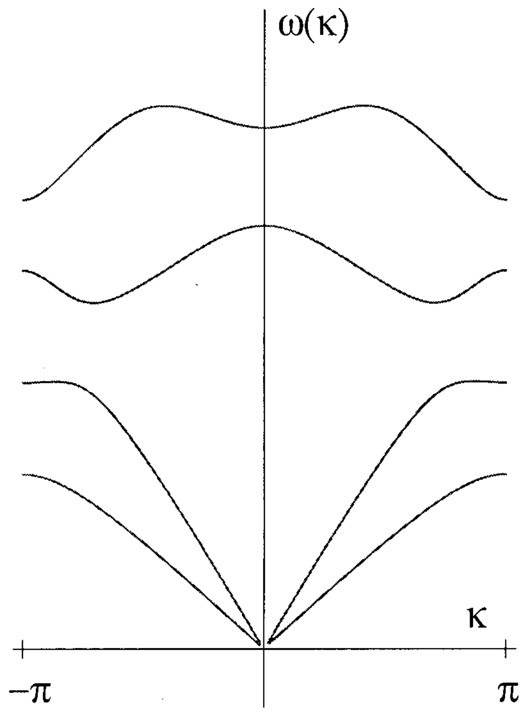

FIG. 2. Reciprocal electromagnetic spectrum of the three-layered periodic stack in Fig. 6 with $\varphi=\pi/2$. The numerical set 1 from Eq. (82); $F/A=2$.

tions of the analysis solely based on symmetry arguments. Indeed, group-theoretical considerations alone can provide only necessary conditions for the desired spectral feature (1), and they cannot substitute for the analysis of real physical mechanisms responsible for the effect (see more on this in [23]).

One of the immediate consequences of spectral nonreciprocity relates to electromagnetic waves with $\vec{k}=0$ belonging to the higher branches of electromagnetic spectrum for which

$$\omega(\vec{0})\neq 0. \tag{6}$$

Indeed, in photonic crystals with reciprocal spectra, the modes with $\vec{k}'=0$ are standing waves and have zero group velocity, as seen in Figs. 1 and 2. For nonreciprocal magnetic photonic crystals the eigenmodes with $\vec{k}=0$ are not standing waves, as one can clearly see in Figs. 3 and 4. For those modes, in every spectral branch (6) the group velocity

$$\vec{u}(\vec{k})=\partial\omega(\vec{k})/\partial\vec{k} \tag{7}$$

does not vanish at $\vec{k}=\vec{0}$, and, consequently, there is a finite energy flux. In fact, the very notion of standing waves in its naive form does not apply to periodic systems with nonreciprocal spectrum. Of course, the property

$$\omega(\vec{k})=\omega(\vec{k}+\vec{b}), \tag{8}$$

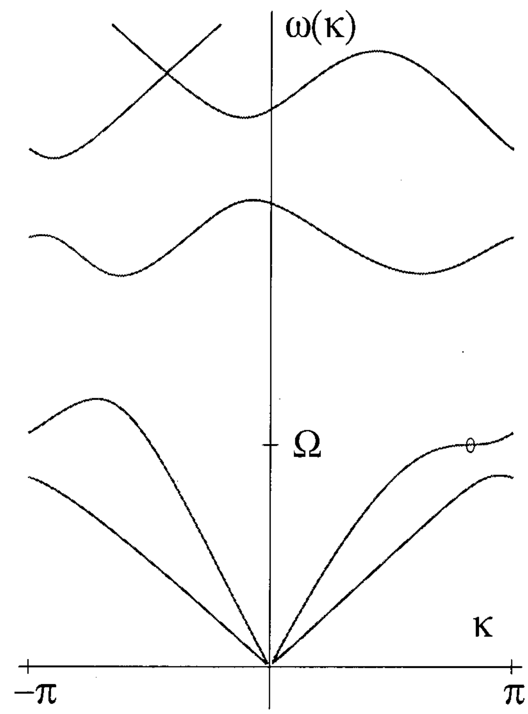

FIG. 3. Nonreciprocal electromagnetic spectrum of the three-layered periodic stack in Fig. 6 with $\varphi=\pi/4$. The numerical set 1 from Eq. (82); $F/A=0.45$. At frequency Ω, one of the spectral branches develops a stationary inflection point.

where $\vec{b}$ is a vector from the reciprocal space, will apply regardless of spectral nonreciprocity.

Consider now a harmonic plane wave propagating along the symmetry direction, say z, of a magnetic photonic crystal, so that both $\vec{u}$ and $\vec{k}$ are parallel to z. Suppose that the spectrum is nonreciprocal, and one of the spectral branches $\omega(k)$ has a stationary inflection point at $k=k_1$ as shown in Fig. 5. By definition, at this point

$$\left.\frac{\partial\omega}{\partial k}\right|_{k=k_1}=0,\quad \left.\frac{\partial^2\omega}{\partial k^2}\right|_{k=k_1}=0,\quad \text{and}\quad \left.\frac{\partial^3\omega}{\partial k^3}\right|_{k=k_1}\neq 0, \tag{9}$$

where $k=k_z$. The spectral branch in Fig. 5 has two Bloch waves with $\omega=\Omega$; the corresponding wave numbers are k_1 and $k_2>k_1$. Obviously, only one of the two waves can transfer the energy: the one with $k=k_2$ and the group velocity $u(k_2)>0$. According to Eq. (9), the backward wave with $k=k_1$ has zero group velocity $u(k_1)=0$ and will not propagate ballistically through the crystal. At first sight, a periodic array with this kind of spectrum will act like a common microwave or optical isolator [24], transmitting radiation only in one of the two opposite directions. But in fact, there is an important difference. An isolator simply eliminates the wave propagating in the ''wrong'' direction whereas the nonreciprocal photonic crystal with stationary inflection point, being transparent for the plane wave with $\omega=\Omega$ and $k=k_2$, ''freezes'' the radiation propagating in the opposite

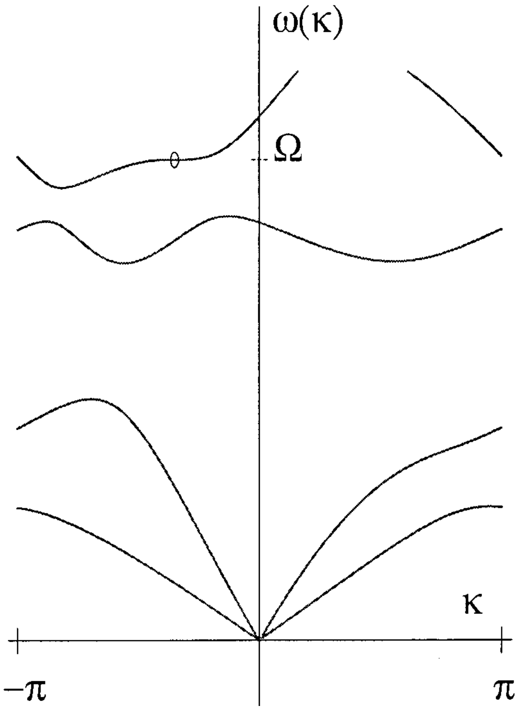

FIG. 4. Nonreciprocal electromagnetic spectrum of the three-layered periodic stack in Fig. 6 with $\varphi=\pi/4$. The numerical set 2 from Eq. (82); $F/A=1.065$. At frequency Ω, one of the spectral branches develops a stationary inflection point.

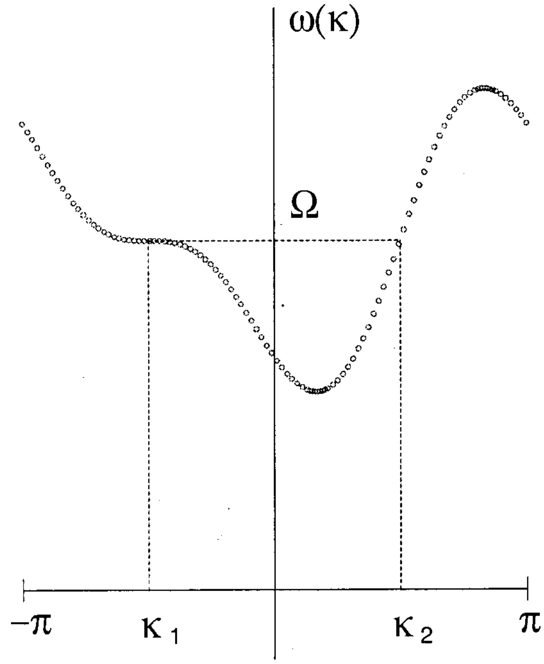

FIG. 5. Asymmetric spectral branch with a stationary inflection point at $\omega=\Omega$, $k=k_1$. Plane waves with $\omega=\Omega$ can only propagate in the positive direction along the z axis.

direction in the form of a *coherent plane wave* with zero group velocity $u=\partial\omega/\partial k$ and its derivative $\partial u/\partial k$.

II. SYMMETRY OF MAGNETIC PHOTONIC CRYSTALS

From a symmetry standpoint, photonic crystals, being spatially periodic, can be viewed as artificial crystals. Therefore, the same symmetry considerations can be applied to electromagnetic waves propagating through photonic crystal, as it is done with electrons, phonons, and photons in crystalline solids (detailed consideration and numerous references can be found in [25–27]). Some examples of a symmetry-based analysis of electromagnetic spectra of nonmagnetic photonic crystals can be found in [1,28,29], and references therein.

The symmetry of a magnetic photonic crystal can be adequately described by its magnetic space group, which along with rotations, reflections, and translations constituting the space symmetry, may also include the time reversal operation $\mathcal{R}$ combined with some space transformations. Magnetic symmetry and its applications in physics of magnetically ordered media have been discussed in numerous textbooks and monographs, including [13], [14], and [19]. Although magnetic symmetry plays an important role in our investigation, we will not engage here in a mathematical analysis requiring the application of the theory of magnetic group representations and corepresentations [25]. This will exclude from our consideration such important problems as spectral degeneracy and compatibility relations in 2D and 3D photonic crystals. Instead, we will focus exclusively on the phenomenon of spectral nonreciprocity (1) and its consequences.

It is well known that eigenmodes associated with any kind of linear waves or excitations in a spatially periodic medium can be chosen in the Bloch form $\psi_{\vec{k}}(\vec{r})$

$$\hat{t}(\vec{a})\,\psi_{\vec{k}}(\vec{r})=\psi_{\vec{k}}(\vec{r}-\vec{a})=\exp(-i\vec{k}\cdot\vec{a})\,\psi_{\vec{k}}(\vec{r}), \qquad (10)$$

where $\hat{t}(\vec{a})$ is the operator of translation by a lattice vector $\vec{a}$.

The space inversion and the time-reversal operations being applied to a Bloch eigenmode $\psi_{\vec{k}}$ change the sign of the Bloch wave vector $\vec{k}$

$$\mathcal{I}\psi_{\vec{k}}=\psi'_{-\vec{k}}, \quad \mathcal{R}\psi_{\vec{k}}=\psi''_{-\vec{k}}. \qquad (11)$$

The first of the two relations (11) immediately follows from Eq. (10). The derivation of the second one can be found, for example, in [27]. An immediate consequence of the relations (11) is that if a periodic medium possesses the space-inversion and/or the time-reversal symmetries, then the Bloch spectrum $\omega(\vec{k})$ of any linear waves and excitations in the medium is reciprocal [27].

If neither $\mathcal{R}$ nor $\mathcal{I}$ is presented in the magnetic symmetry group of the photonic crystal, still it may be some other symmetry operations that ensure the spectral reciprocity (5). In what follows we will use a simple sufficient condition for the spectral reciprocity introduced in [18]: If the symmetry group G of a periodic structure includes a symmetry operation g that changes the sign of the Bloch vector $\vec{k}$, then the spectrum $\omega(\vec{k})$ of Bloch eigenmodes supported by the system will be reciprocal for this particular direction of $\vec{k}$

$$\text{if } \hat{g}\vec{k}=-\vec{k} \quad \text{then } \{\omega_n(\vec{k})\}=\{\omega_n(-\vec{k})\}, \qquad (12)$$

where n indexes spectral branches and $\omega_n(\vec{k})$ is the respective dispersion relation. In particular, if $\mathcal{R}\in G$ and/or $\mathcal{I}\in G$, the reciprocity criterion (12) is met automatically for all directions of wave propagation, since as it follows from Eq. (11):

$$\mathcal{I}\vec{k}=-\vec{k},\quad \mathcal{R}\vec{k}=-\vec{k} \tag{13}$$

[compare these relations with Eq. (3)].

So, when looking for the spectral nonreciprocity (1), one has to be sure that the magnetic symmetry group G of the periodic structure does not include any operations that change the sign of the wave vector $\vec{k}$ [18],

$$\hat{g}\vec{k}\neq-\vec{k}\quad \text{for any}\ \ g\in G. \tag{14}$$

Whether or not the criterion (14) is met may depend on the direction of the wave vector $\vec{k}$, unless $\mathcal{R}\in G$ and/or $\mathcal{I}\in G$. The expression (14) is just a necessary condition for spectral asymmetry. Even if this condition is met, the corresponding effect of spectral nonreciprocity may appear to be negligible or even ruled out by some physical reasons different from magnetic symmetry restrictions. Yet the criterion (14) allows us to substantially narrow down the search area. To find out whether a periodic system satisfying this criterion does display the spectral nonreciprocity one has to go beyond the symmetry consideration, which will be done in the Secs. IV–VI.

If none of the constitutive components of a photonic crystal supports any kind of spontaneous magnetic order and there is no external magnetic field, then the photonic crystal certainly possesses the time-reversal symmetry $\mathcal{R}$ and, therefore, will support a reciprocal spectrum.

If, on the other hand, an external magnetic field $\vec{H}_0$ is applied, and/or at least one of the constitutive components supports a spontaneous magnetic order, then the medium may not support the time-reversal symmetry $\mathcal{R}$ [13]. At the same time, if the space-inversion symmetry $\mathcal{I}$ is in place (as is the case in Fig. 7), the Bloch spectrum $\omega(\vec{k})$ still will be reciprocal. To the best of our knowledge, this has been the case with all magnetic photonic crystals considered in the literature.

III. MAGNETIC SYMMETRY OF PERIODIC STACKS

In this section we apply the symmetry criterion (14) to 1D magnetic photonic crystals (periodic magnetic stacks). It will enable us to develop some important guidelines for designing magnetic photonic crystals with spectral nonreciprocity.

A. Choice of constitutive components

Our objective here is to find the simplest periodic arrays of dielectric layers satisfying the necessary condition (14) for spectral nonreciprocity. With this in mind, let us restrict our consideration to periodic stacks composed of just two different sorts of layers, which will be referred to as the $\mathcal{A}$ layers and the $\mathcal{F}$ layers. Some examples are presented in Figs. 6, 7, and 8.

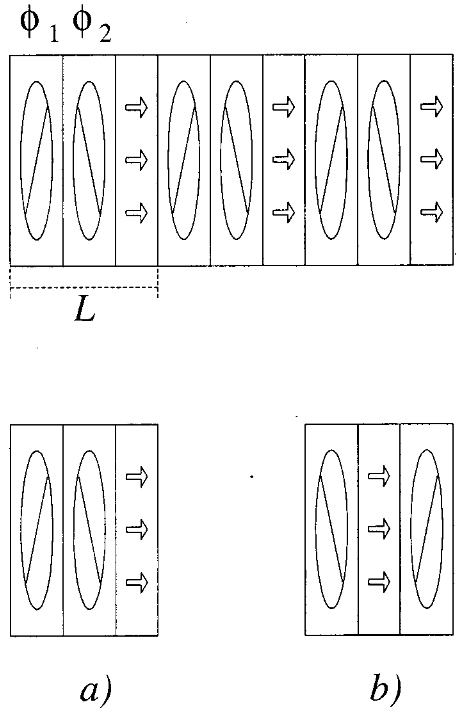

FIG. 6. A fragment of a three-layered periodic array composed of the $\mathcal{A}$ and $\mathcal{F}$ layers. The arrows show the direction of the magnetic polarization $\vec{M}_0$ of the $\mathcal{F}$ layers. $L=2A+F$—the primitive translation; φ_1 and φ_2—orientation angles of the $\mathcal{A}$ layers. The fragments (a) and (b) represent two different choices of a primitive cell.

(i) The $\mathcal{A}$ layers are made of a nonmagnetic dielectric material with anisotropy in the xy plane

$$\hat{\varepsilon}_{\mathcal{A}}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & 0\\ \varepsilon_{xy} & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz}\end{bmatrix} =\begin{bmatrix}\varepsilon+\delta\cos 2\varphi & \delta\sin 2\varphi & 0\\ \delta\sin 2\varphi & \varepsilon-\delta\cos 2\varphi & 0\\ 0 & 0 & \varepsilon_{zz}\end{bmatrix}, \tag{15}$$

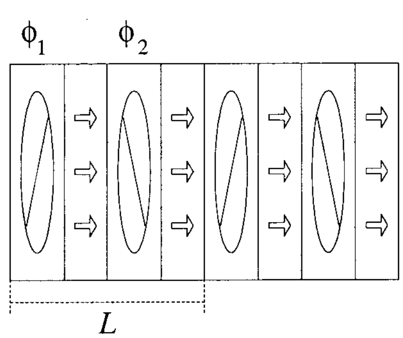

FIG. 7. A fragment of a four-layered periodic array with a parallel arrangement of the ferromagnetic layers. Each magnetic layer is sandwiched between two anisotropic ones, and vice versa. The magnetic symmetry $m'm'm$ ensures spectral reciprocity $\omega(\vec{k})=\omega(-\vec{k})$ for an arbitrary direction $\vec{k}$ of wave propagation.

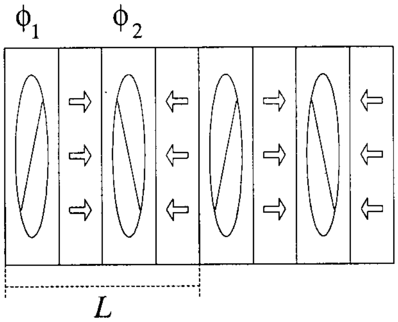

FIG. 8. A fragment of a four-layered periodic array with an antiparallel arrangement of the ferromagnetic layers. Each magnetic layer is sandwiched between two anisotropic ones and vice versa. The magnetic symmetry mmm' is compatible with spectral nonreciprocity $\omega(\vec{k})\neq\omega(-\vec{k})$ unless $\vec{k}\perp z$.

$$\hat{\mu}_{\mathcal{A}}=\hat{1}.$$

All components of the tensors $\hat{\varepsilon}_{\mathcal{A}}$ are presumed real. Parameter δ describes the magnitude of in-plane anisotropy, while the angle φ defines the orientation of the principle axes of tensor $\hat{\varepsilon}_{\mathcal{A}}$ in the *xy* plane. The orientation φ may vary from layer to layer. All $\mathcal{A}$ layers are made up of the same dielectric material (15) and have the same thickness *A*. The only parameter that may differ in different $\mathcal{A}$ layers is the angle φ.

(ii) The $\mathcal{F}$ layers are ferromagnetic with magnetization $\vec{M}_0$ parallel to the *z* direction; there is no in-plane anisotropy in this case

$$\hat{\varepsilon}_{\mathcal{F}}=\begin{bmatrix}\varepsilon & i\alpha & 0\\ -i\alpha & \varepsilon & 0\\ 0 & 0 & \varepsilon_{zz}\end{bmatrix},\quad \hat{\mu}_{\mathcal{F}}=\begin{bmatrix}\mu & i\beta & 0\\ -i\beta & \mu & 0\\ 0 & 0 & \mu_{zz}\end{bmatrix}. \tag{16}$$

The real parameters α and β are responsible for the magnetic Faraday rotation [13,30]. All $\mathcal{F}$ layers have the same thickness *F*.

The anisotropic dielectric material of the $\mathcal{A}$ layers, as well as the magnetically polarized material of the $\mathcal{F}$ layers, both possess the space-inversion symmetry $\mathcal{I}$. Hence, each of the two materials, if they fill the entire space, will support perfectly reciprocal electromagnetic spectra. In such a case, the spectral nonreciprocity of the periodic stack, if it occurs, will be essentially related to the geometry of the periodic structure.

The restrictions (15) and (16) imposed on the constitutive components of magnetic photonic crystals can be justified as follows. Firstly, dielectric materials with property tensors (15) and (16) are readily available, their electromagnetic properties are predictable and well established. Secondly, the incorporation of dielectric materials with more complex or more diverse physical properties presumably will not render any qualitatively new spectral features, compared to what one can achieve with just the $\mathcal{A}$ and $\mathcal{F}$ layers. On the other hand, as shown in Sec. III B, we do need both the ferromagnetic layers (the $\mathcal{F}$ layers) and the layers with alternate in-plane anisotropy (the $\mathcal{A}$ layers) to achieve an appreciable spectral asymmetry. Hence, it seems to be impossible to further simplify the composition of 1D magnetic photonic crystals without losing the desirable electromagnetic properties.

At this point we would like to recall some basic facts relating to the ferromagnetic media. For systematic consideration see, for instance, [25], [31], and [13].

Magnetic field $\vec{\mathbf{H}}$ and magnetization $\vec{\mathbf{M}}$ within the $\mathcal{F}$ layers are superpositions of two contributions

$$\vec{\mathbf{H}}=\vec{H}_0+\vec{H},\quad \vec{\mathbf{M}}=\vec{M}_0+\vec{M}. \tag{17}$$

Here $\vec{H}_0$ and $\vec{M}_0$ are the static components of field and magnetization, while $\vec{H}$ and $\vec{M}$ are the alternating components, associated with the linear monochromatic electromagnetic wave (see footnote 1). Throughout this paper we deal with linearized Maxwell equations (35) and linearized constitutive relations (4), written in terms of small harmonic variables $\vec{E},\vec{H},\vec{D},\vec{B}$. The role of the finite static field $\vec{H}_0$ and magnetization $\vec{M}_0$ reduces to the formation of the material tensors (16) related to the $\mathcal{F}$ layers. In particular, the gyrotropic parameters α and β in Eq. (16) essentially depend on $\vec{H}_0$, as well as on the frequency ω. At the microwave frequency range, the dominant contribution to the Faraday rotation comes from the ''magnetic'' parameter β, which can become particularly large in the vicinity of ferromagnetic resonance. The diagonal element μ in Eq. (16) is also sensitive to $\vec{H}_0$ and ω. The substitution

$$\vec{H}_0\to-\vec{H}_0\quad\text{and}\quad\vec{M}_0\to-\vec{M}_0 \tag{18}$$

implies the following transformation of $\hat{\varepsilon}_{\mathcal{F}}$ and $\hat{\mu}_{\mathcal{F}}$ in Eq. (16):

$$\varepsilon\to\varepsilon,\quad \mu\to\mu,\quad \alpha\to-\alpha,\quad \beta\to-\beta \tag{19}$$

which reduces to the change of sign of the Faraday rotation.

In magnetically soft ferrites and ferromagnets, the direction of static magnetization $\vec{M}_0$ coincides with the direction of static magnetic field $\vec{H}_0$. Specifically, in the case of interest

$$\vec{H}_0\|\vec{M}_0\|z. \tag{20}$$

In such a case, all $\mathcal{F}$ layers of the periodic array are exactly identical.

In the opposite case of ferromagnets with strong uniaxial anisotropy (see, for example, [31]), the orientation $\vec{M}_0\|z$ can be sustained even without the external field $\vec{H}_0\|z$. Therefore, one can arrange a periodic magnetic stack with an antiparallel orientation of magnetic polarization $\vec{M}_0$ in the neighboring $\mathcal{F}$ layers, as shown in Fig. 8. Notice that the stability of the antiparallel orientation of the magnetizations $\vec{M}_0$ in different $\mathcal{F}$ layers requires a relatively strong uniaxial anisotropy of the ferromagnetic material. Besides, even a small external magnetic field would cause the violation of interrelations (19) for $\mathcal{F}$ layers with an antiparallel orientation of

$\vec{M}_0$. Therefore, when dealing with an antiparallel arrangement of the $\mathcal{F}$ layers we will automatically assume that $\vec{H}_0 = 0$.

In future consideration we, for the most part, will be dealing with the parallel orientation (20) of the $\mathcal{F}$ layers. In such a case, all $\mathcal{F}$ layers of the periodic stack are identical regardless of the magnitude of the static magnetic field $\vec{H}_0 \| z$.

Let us introduce the following notations for the $\mathcal{A}$ and $\mathcal{F}$ layers, as well as for the periodic stacks made up of these layers. The symbol

$$\begin{bmatrix} \mathcal{A} \\ \varphi \end{bmatrix} \tag{21}$$

denotes an individual $\mathcal{A}$ layer with the orientation angle φ. The symbols

$$\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix} \text{ and } \begin{bmatrix} \mathcal{F} \\ - \end{bmatrix} \tag{22}$$

denote individual $\mathcal{F}$ layers with two opposite directions of magnetization $\vec{M}_0 \| z$.

Using the notations (21) and (22), a primitive cell of the three-layered periodic stack in Fig. 6 can be represented as

$$\begin{bmatrix} \mathcal{A} \\ \varphi_1 \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_2 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix} \tag{23}$$

or, equivalently,

$$\begin{bmatrix} \mathcal{A} \\ \varphi_2 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_1 \end{bmatrix} \equiv \begin{bmatrix} \mathcal{F} \\ + \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_1 \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_2 \end{bmatrix}. \tag{24}$$

The expressions (23) and (24) correspond to different choices of a primitive cell of the same periodic array.

For two four-layered periodic stacks shown in Figs. 7 and 8 we, respectively, have

$$\begin{bmatrix} \mathcal{A} \\ \varphi_1 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_2 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix} \tag{25}$$

and

$$\begin{bmatrix} \mathcal{A} \\ \varphi_1 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ + \end{bmatrix}\begin{bmatrix} \mathcal{A} \\ \varphi_2 \end{bmatrix}\begin{bmatrix} \mathcal{F} \\ - \end{bmatrix}. \tag{26}$$

The expressions (23)–(26) define the layers' arrangement within a single primitive cell and, thereby, define the entire periodic arrays.

B. Space arrangement of the layers

Recall that a 1D photonic crystal is a periodic array of identical fragments (primitive cells). Each primitive cell includes several $\mathcal{A}$ and $\mathcal{F}$ layers. The choice of a primitive cell for a given periodic stack is not unique. The magnetic symmetry of the entire 1D magnetic photonic crystal is determined by the following geometric factors relating to a given primitive cell: (i) the number of $\mathcal{A}$ and $\mathcal{F}$ layers in a cell and their arrangement; (ii) the relative orientations φ_{ij} of the $\mathcal{A}$ layers.[2]

Suppose that $\vec{k} \| z$. Let m denote the reflection in a mirror plane containing the z axis, and let m' denote the reflection m accompanied by the time-reversal operation $\mathcal{R}$,

$$m' \equiv \mathcal{R}m \equiv m\mathcal{R}.$$

The operation m does not change $k = k_z$, whereas m' transforms k into $-k$ [see formula (13)]. The following statement determines the minimal complexity of a periodic stack, necessary for spectral nonreciprocity.

Let $\varphi_{ij} = \varphi_i - \varphi_j$ be the misalignment angle between a pair of $\mathcal{A}$ layers i and j. If each φ_{ij} in a primitive cell is a multiple of $\pi/2$ then the magnetic group G of the stack includes the symmetry operation m' transforming k into $-k$ and, thereby, implying spectral reciprocity.

The above statement implies that a 1D photonic crystal may support a nonreciprocal electromagnetic spectrum only if its primitive cell includes at least one $\mathcal{F}$ layer and at least two $\mathcal{A}$ layers with the misalignment angle $\varphi_{12} = \varphi_1 - \varphi_2$ different from 0 and $\pi/2$.

Note that periodic stacks with just two layers in a primitive cell will always support the reciprocal electromagnetic spectrum *regardless of the direction of the wave vector* $\vec{k}$. Indeed, in this latter case a 1D photonic crystal certainly possesses the space inversion symmetry $\mathcal{I}$. For more on the case of an off-axis wave propagation see Sec. III C.

1. Periodic stacks with a three-layered cell

Let us consider the case of a three-layered primitive cell in more detail. There is only one essentially different periodic array of the $\mathcal{A}$ and $\mathcal{F}$ layers with three layers in a primitive cell. It can be chosen as presented in Eq. (23) and Fig. 6. A primitive cell is comprised of one $\mathcal{F}$ layer and two $\mathcal{A}$ layers with different orientations φ_1 and φ_2. The most critical parameter of this periodic structure is the misalignment angle

$$\varphi = \varphi_2 - \varphi_1$$

between the adjacent $\mathcal{A}$ layers. The angle φ must not be a multiple of $\pi/2$

$$\varphi = \varphi_2 - \varphi_1 \neq 0, \quad \pi/2 \tag{27}$$

(see the previously made remark[2]). If the condition (27) is met, the magnetic symmetry group G of the periodic array is

$$G \equiv 2'2'2 \quad \text{for } \varphi \neq 0, \pi/2. \tag{28}$$

[2]The orientations φ and $\varphi + \pi$ are identical. Hence, there is no difference between, for instance, $\varphi = 0$ and $\varphi = \pi$, or between $\varphi = +\pi/2$ and $\varphi = -\pi/2$.

None of the four symmetry operations from $2'2'2$ transforms $\vec{k} \to -\vec{k}$ for $\vec{k} \| z$, therefore, the configuration (27) meets the necessary condition (14) of spectral nonreciprocity.

If the condition (27) is not met, then the magnetic symmetry group G of the periodic array (23) raises up to

$$G \equiv \bar{4} m' m' \quad \text{for} \quad \varphi = \pi/2 \tag{29}$$

or

$$G \equiv m' m' m \quad \text{for} \quad \varphi = 0. \tag{30}$$

The symmetry (29) implies spectral reciprocity for $\vec{k} \| z$, while the symmetry group (30) ensures the spectral reciprocity $\omega(\vec{k}) = \omega(-\vec{k})$ for arbitrary $\vec{k}$.

In further consideration we assume that the condition (27) is met, unless otherwise specified. The computation of electromagnetic spectra carried out in the following sections shows that strong spectral nonreciprocity does occur in the periodic stacks when the necessary condition (27) is satisfied.

2. Periodic stacks with four-layered cell

First, let us consider a simple augmentation (25) to the three-layered structure (23). Each magnetic layer now is sandwiched between two anisotropic ones, and vice versa. The magnetic symmetry

$$G \equiv m' m' m \tag{31}$$

of this periodic array appears to be higher than that of the three-layered structure (23). In fact, the magnetic group $m' m' m$ even includes the space inversion $\mathcal{I}$ and, therefore, ensures the spectral reciprocity regardless of the direction $\vec{k}$ of the light propagation, and regardless of the value of the misalignment angle φ.

Consider now the four-layered periodic stack (26) with an antiparallel arrangement of the neighboring ferromagnetic layers. If the condition (27) is met, the magnetic symmetry group of the four-layered stack in Fig. 8 is

$$G \equiv m m m' \tag{32}$$

which is compatible with spectral nonreciprocity $\omega(\vec{k}) \neq \omega(-\vec{k})$, unless $\vec{k} \perp z$. The periodic array (26) in Fig. 8 has zero bulk magnetic polarization, because the contributions of the individual ferromagnetic layers to the bulk magnetization cancel out within each primitive cell. The absence of bulk magnetization results in the absence of the related nonuniform demagnetization fields within the stack [32]. This feature may be of great advantage when the magnetic field uniformity within a stack is critical. On the other hand, such an array cannot be stabilized by an external magnetic field $\vec{H}_0 \| z$, and therefore, requires a ferromagnetic material with sufficient uniaxial magnetic anisotropy.

C. Off-axis wave propagation

Let us now consider a situation when the magnetic symmetry group G of a periodic stack includes neither the space inversion $\mathcal{I}$ nor the time-reversal operations $\mathcal{R}$. At the same time, suppose that the criterion (12) of the spectral reciprocity is satisfied at least for $\vec{k} \| z$. This implies that for the particular direction $\vec{k} \| z$ of the wave propagation, the electromagnetic spectrum must be reciprocal, i.e.,

$$\text{if} \quad \vec{k} \| z \quad \text{then} \quad \omega(\vec{k}) = \omega(-\vec{k}). \tag{33}$$

A simple example of the kind is provided by the three-layered configuration depicted in Fig. 6 with $\varphi = \pi/2$. Indeed, the magnetic symmetry group (29) of this array does include the elements

$$\bar{4} \equiv 4_z m_z \quad \text{and} \quad m'_x \equiv m_x \mathcal{R},$$

each of which transforms $k_z \to -k_z$ and thereby ensures the spectral reciprocity (33). On the other hand, if $\vec{k} \nparallel z$ then it is possible that none of the symmetry operations from Eq. (29) changes $\vec{k} \to -\vec{k}$ and, therefore, the electromagnetic spectrum may not be reciprocal. Thus, in some cases, the spectral nonreciprocity can occur only when the wave vector $\vec{k}$ deviates from the distinguished direction z. Of course, it may occur only if the magnetic symmetry group G of photonic crystal includes neither the space inversion nor the time reversal. For example, this cannot be the case with the three-layered configuration in Fig. 6 with $\varphi = 0$,[3] or with the four-layered configuration (25) in Fig. 7 regardless of the value of φ.

IV. TRANSFER MATRIX OF A MAGNETIC STACK

This and the following two sections are devoted to the quantitative analysis of electromagnetic spectra of different 1D magnetic periodic arrays composed of the $\mathcal{A}$ and the $\mathcal{F}$ layers. In Secs. IV and V we reformulate the transfer matrix formalism for the case of magnetic stratified media with the possibility of spectral nonreciprocity. Then, in Sec. VI we consider several specific numerical examples, which will prove the existence of strong spectral nonreciprocity in the situations predicted by the symmetry analysis of the preceding two sections.

Let us consider a plane monochromatic wave propagating along the z direction in a 1D periodic stack with the constitutive components defined in Eqs. (15) and (16). For this geometry all the alternating components of the electromagnetic wave are perpendicular to the z direction

[3] In the case $\varphi = 0$, the three-layered configuration (23) reduces to a two-layered one with the doubled effective thickness of the $\mathcal{A}$ layers.

$$\vec{E},\vec{H},\vec{D},\vec{B}\perp\vec{z}, \tag{34}$$

and are independent of the x and y coordinates. The Maxwell equations

$$\vec{\nabla}\times\vec{E}=\frac{i\omega}{c}(\vec{B}),\quad \vec{\nabla}\times\vec{H}=-\frac{i\omega}{c}(\vec{D}), \tag{35}$$

then can be recast as

$$\hat{\sigma}\frac{\partial}{\partial z}\vec{E}=\frac{i\omega}{c}(\hat{\mu}\vec{H}),\quad \hat{\sigma}\frac{\partial}{\partial z}\vec{H}=-\frac{i\omega}{c}(\hat{\varepsilon}\vec{E}), \tag{36}$$

where

$$\vec{E}(z)=\begin{bmatrix}E_x(z)\\E_y(z)\end{bmatrix},\quad \vec{H}(z)=\begin{bmatrix}H_x(z)\\H_y(z)\end{bmatrix},$$

$$\hat{\sigma}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}=-\hat{\sigma}^{-1}. \tag{37}$$

At layers' interfaces, where the tensors $\hat{\varepsilon}=\hat{\varepsilon}(z)$ and/or $\hat{\mu}=\hat{\mu}(z)$ are discontinuous, the following four interface conditions must be satisfied:

$$\begin{bmatrix}\vec{E}(z_m-0)\\\vec{H}(z_m-0)\end{bmatrix}=\begin{bmatrix}\vec{E}(z_m+0)\\\vec{H}(z_m+0)\end{bmatrix}. \tag{38}$$

Here $z=z_m$ is the position of the interface between the $(m-1)$th and mth layers.

A. The transfer matrix

Various modifications of the transfer matrix formalism have been widely used for the analysis of electromagnetic properties of stratified media (see, for instance, [30,9,11,3]). We will use similar formalisms to analyze periodic stacks with spectral nonreciprocity.

Notice that the reduced 1D Maxwell equations (36) constitute a system of four ordinary linear differential equations of the first order. Its general monochromatic solution involves four arbitrary coefficients that can be uniquely related to the four quantities

$$\begin{bmatrix}\vec{E}(z_1)\\\vec{H}(z_1)\end{bmatrix} \tag{39}$$

which are the transverse field components at a fixed point z_1. The monochromatic electromagnetic field at an arbitrary point z_2 can be related to its value at $z=z_1$ at the same point in time by the following linear relation:

$$\begin{bmatrix}\vec{E}(z_2)\\\vec{H}(z_2)\end{bmatrix}=\hat{\mathbf{T}}(z_2,z_1)\begin{bmatrix}\vec{E}(z_1)\\\vec{H}(z_1)\end{bmatrix}, \tag{40}$$

where a 4×4 matrix $\hat{\mathbf{T}}(z_2,z_1)$ is called a *transfer matrix*. The coordinates z_2 and z_1 may include an external boundary (or boundaries) of the dielectric slab, as well as layers' interfaces. According to Eq. (38), both vector quantities $\vec{E}(z)$ and $\vec{H}(z)$ are continuous functions of z even at the points of the property tensors discontinuity. Therefore, the matrix $\hat{\mathbf{T}}(z_2,z_1)$ is also a continuous function of z_2 and z_1.

The following basic property of the transfer matrix is a direct consequence of its definition (40)

$$\hat{\mathbf{T}}(z_2;z_1)=\hat{\mathbf{T}}(z_2;z)\hat{\mathbf{T}}^{-1}(z_1;z)=\hat{\mathbf{T}}^{-1}(z_1;z_2). \tag{41}$$

In the case of a stratified medium, one can introduce the transfer matrix $\hat{\mathbf{T}}_m$ for every individual layer m. By definition, a single-layer transfer matrix $\hat{\mathbf{T}}_m$ relates the electromagnetic field components at two faces of the layer at the same point in time

$$\begin{bmatrix}\vec{E}(z_{m+1})\\\vec{H}(z_{m+1})\end{bmatrix}=\hat{\mathbf{T}}_m\begin{bmatrix}\vec{E}(z_m)\\\vec{H}(z_m)\end{bmatrix}. \tag{42}$$

The coordinates z_m and z_{m+1} correspond to the left and right faces of the mth layer. We note that $\vec{E}(z)$ and $\vec{H}(z)$ are continuous at the layer interfaces. The transfer matrix $\hat{\mathbf{T}}_S$ of a stack of layers is the sequential product of all one-layer matrices $\hat{\mathbf{T}}_m$,

$$\hat{\mathbf{T}}_S=\prod_m \hat{\mathbf{T}}_m. \tag{43}$$

Different matrices $\hat{\mathbf{T}}_m$ of the product may not commute. The explicit expressions for the single-layer transfer matrices $\hat{\mathbf{T}}_m$ will be obtained in the next section.

B. Single-layer transfer matrices

Within a single uniform layer, the monochromatic solutions for the Maxwell equations (36) are harmonic plane waves

$$\vec{E}(z)=\vec{E}\exp(iqz),\quad \vec{H}(z)=\vec{H}\exp(iqz), \tag{44}$$

where q is the in-layer wave vector. The eigenvectors $\vec{E}$ and $\vec{H}$ are defined by the equation

$$n\hat{\sigma}\vec{E}-\hat{\mu}\vec{H}=0,\quad \hat{\varepsilon}\vec{E}+n\hat{\sigma}\vec{H}=0\quad \text{where}\quad n=\frac{cq}{\omega}. \tag{45}$$

The system (45) of the four linear equations describes four electromagnetic eigenmodes within the layer

$$\begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix} = e^{iq_1 z} \begin{bmatrix} \vec{E}_1 \\ \vec{H}_1 \end{bmatrix}, \quad \begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix} = e^{-iq_1 z} \begin{bmatrix} \vec{E}_1 \\ -\vec{H}_1 \end{bmatrix},$$
$$\begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix} = e^{iq_2 z} \begin{bmatrix} \vec{E}_2 \\ \vec{H}_2 \end{bmatrix}, \quad \begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix} = e^{-iq_2 z} \begin{bmatrix} \vec{E}_2 \\ -\vec{H}_2 \end{bmatrix}, \tag{46}$$

where

$$q_1 = \frac{\omega}{c} n_1 = \frac{\omega}{c}\sqrt{\eta_1}, \quad q_2 = \frac{\omega}{c} n_2 = \frac{\omega}{c}\sqrt{\eta_2}, \tag{47}$$

and η_1 and η_2 are the eigenvalues of the tensor

$$\hat{\eta} = -\hat{\sigma}\hat{\mu}\hat{\sigma}\hat{\varepsilon}. \tag{48}$$

In general, if the Hermitian matrices $\hat{\varepsilon}$ and $\hat{\mu}$ do not commute, the tensor $\hat{\eta} = -\hat{\sigma}\hat{\mu}\hat{\sigma}\hat{\varepsilon}$ is not Hermitian and the complex vectors $\vec{E}_1$ and $\vec{E}_2$ may not be orthogonal.

Let us turn to the transfer matrix within a uniform layer. From the definition (40) one can see that the four in-layer solutions (46) for the Maxwell equations are the eigenvectors of the in-layer transfer matrix $\hat{\mathbf{T}}(z) \equiv \hat{\mathbf{T}}(z,0)$,

$$\hat{\mathbf{T}}(z) \begin{bmatrix} \vec{E}_1 \\ \vec{H}_1 \end{bmatrix} = e^{iq_1 z} \begin{bmatrix} \vec{E}_1 \\ \vec{H}_1 \end{bmatrix}, \quad \hat{\mathbf{T}}(z) \begin{bmatrix} \vec{E}_1 \\ -\vec{H}_1 \end{bmatrix} = e^{-iq_1 z} \begin{bmatrix} \vec{E}_1 \\ -\vec{H}_1 \end{bmatrix},$$
$$\hat{\mathbf{T}}(z) \begin{bmatrix} \vec{E}_2 \\ \vec{H}_2 \end{bmatrix} = e^{iq_2 z} \begin{bmatrix} \vec{E}_2 \\ \vec{H}_2 \end{bmatrix}, \quad \hat{\mathbf{T}}(z) \begin{bmatrix} \vec{E}_2 \\ -\vec{H}_2 \end{bmatrix} = e^{-iq_2 z} \begin{bmatrix} \vec{E}_2 \\ -\vec{H}_2 \end{bmatrix}. \tag{49}$$

Using Eq. (49) one can express the transfer matrix $\hat{\mathbf{T}}(z)$ in terms of the eigenvectors $\vec{E}_1$ and $\vec{H}_1$ from Eq. (46),

$$\hat{\mathbf{T}}(z) = \hat{\mathbf{T}}(z_2 - z_1) = \hat{\mathbf{W}}(z_2)\hat{\mathbf{W}}^{-1}(z_1), \tag{50}$$

where the matrix

$$\hat{\mathbf{W}}(z) = \begin{bmatrix} E_{1,x}e^{iq_1 z} & E_{1,x}e^{-iq_1 z} & E_{2,x}e^{iq_2 z} & E_{2,x}e^{-iq_2 z} \\ E_{1,y}e^{iq_1 z} & E_{1,y}e^{-iq_1 z} & E_{2,y}e^{iq_2 z} & E_{2,y}e^{-iq_2 z} \\ H_{1,x}e^{iq_1 z} & -H_{1,x}e^{-iq_1 z} & H_{2,x}e^{iq_2 z} & -H_{2,x}e^{-iq_2 z} \\ H_{1,y}e^{iq_1 z} & -H_{1,y}e^{-iq_1 z} & H_{2,y}e^{iq_2 z} & -H_{2,y}e^{-iq_2 z} \end{bmatrix} \tag{51}$$

is composed of the Cartesian components of the eigenvectors $\vec{E}_1$ and $\vec{H}_1$ from Eq. (46).

Along with the basic property (41), the transfer matrix $\hat{\mathbf{T}}(z)$ from Eq. (50) displays the following important symmetry:

$$\hat{\mathbf{T}}(z) = \hat{\mathbf{T}}(z_2 - z_1) = \hat{\mathbf{T}}^{-1}(-z), \tag{52}$$

which is an immediate consequence of the substance uniformity. Another general consequence of the uniformity is that the eigenvectors of the transfer matrix $\hat{\mathbf{T}}(z)$ coincide with the solutions (46) for the Maxwell equations within the layer.

According to Eq. (49), $\hat{\mathbf{T}}(z)$ is equivalent to a unitary matrix and, in particular,

$$\det[\hat{\mathbf{T}}(z)] = 1. \tag{53}$$

Notice that the property (53) does not apply to dielectric materials with linear magnetoelectric response (2). This and related questions are discussed in the Appendix.

1. Transfer matrix for a single $\mathcal{A}$ layer

The explicit expression for a single-layer transfer matrix can be obtained from Eqs. (50) and (51) by substitution of the explicit expressions for the eigenvectors $\vec{E}_1$, $\vec{H}_1$, $\vec{E}_2$, and $\vec{H}_2$. Those eigenvectors, in turn, are the solutions for the system (45), into which the corresponding property tensors $\hat{\varepsilon}$ and $\hat{\mu}$ should be plugged in.

In the case of $\mathcal{A}$ layers, the property tensors are defined in Eq. (15). The substitution of Eq. (15) into Eq. (45) yields

$$\vec{E}_1 = \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix}, \quad \vec{H}_1 = n_1 \begin{bmatrix} -\sin\varphi \\ \cos\varphi \end{bmatrix},$$
$$\vec{E}_2 = \begin{bmatrix} -\sin\varphi \\ \cos\varphi \end{bmatrix}, \quad \vec{H}_2 = n_2 \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \end{bmatrix}, \tag{54}$$

where

$$n_1 = \sqrt{\varepsilon + \delta}, \quad n_2 = \sqrt{\varepsilon - \delta}. \tag{55}$$

This gives us the following explicit expression for the transfer matrix $\hat{\mathbf{T}}_A(\varphi)$ of the $\mathcal{A}$ layer of the thickness A and orientation φ,

$$\hat{\mathbf{T}}_A(\varphi)=\begin{bmatrix} u^2u_1+v^2u_2 & uvu_1-uvu_2 & -\frac{iuvv_1}{n_1}+\frac{iuvv_2}{n_2} & \frac{iu^2v_1}{n_1}+\frac{iv^2v_2}{n_2} \\ uvu_1-uvu_2 & v^2u_1+u^2u_2 & -\frac{iv^2v_1}{n_1}-\frac{iu^2v_2}{n_2} & \frac{iuvv_1}{n_1}-\frac{iuvv_2}{n_2} \\ -in_1uvv_1+in_2uvv_2 & -in_1v^2v_1-in_2u^2v_2 & v^2u_1+u^2u_2 & -uvu_1+uvu_2 \\ in_1u^2v_1+in_2v^2v_2 & in_1uvv_1-in_2uvv_2 & -uvu_1+uvu_2 & u^2u_1+v^2u_2 \end{bmatrix}, \tag{56}$$

where

$$\begin{gathered} u_1=\cos(q_1A)=\cos(n_1a), \quad v_1=\sin(q_1A)=\sin(n_1a), \\ u_2=\cos(q_2A)=\cos(n_2a), \quad v_2=\sin(q_2A)=\sin(n_2a), \\ u=\cos\varphi, \quad v=\sin\varphi. \end{gathered} \tag{57}$$

Notice that the dependence of the transfer matrix $\hat{\mathbf{T}}_A(\varphi)$ in Eq. (57) on the wave frequency ω and the layer thickness A comes through a single dimensionless parameter

$$a=\frac{\omega}{c}A.$$

There are three more independent dimensionless physical parameters in the expression for $\hat{\mathbf{T}}_A(\varphi)$: two refractive indices n_1 and n_2 defined in Eq. (55), and the angle φ of the layer orientation defined in Eq. (15). Evidently

$$\hat{\mathbf{T}}_A(\varphi)=\hat{\mathbf{T}}_A(\varphi+\pi). \tag{58}$$

2. Transfer matrix for a single $\mathcal{F}$ layer

The derivation of the transfer matrix for an $\mathcal{F}$ layer is similar to that for an $\mathcal{A}$ layer. Instead of Eq. (54), within an $\mathcal{F}$ layer we have the following expressions for the eigenvectors $\vec{E}_1$, $\vec{H}_1$, $\vec{E}_2$, and $\vec{H}_2$:

$$\vec{E}_1=\begin{bmatrix}1\\-i\end{bmatrix}, \quad \vec{H}_1=\gamma_1\begin{bmatrix}i\\1\end{bmatrix}, \quad \vec{E}_2=\begin{bmatrix}-i\\1\end{bmatrix}, \quad \vec{H}_2=\gamma_2\begin{bmatrix}-1\\-i\end{bmatrix}, \tag{59}$$

where

$$\gamma_1=\sqrt{(\varepsilon+\alpha)(\mu+\beta)^{-1}}, \quad \gamma_2=\sqrt{(\varepsilon-\alpha)(\mu-\beta)^{-1}}. \tag{60}$$

The substitution of Eq. (59) into Eqs. (51) and (50) produces the following expression for the transfer matrix $\hat{\mathbf{T}}_{\mathcal{F}}$ of a uniaxial ferromagnetic layer of the thickness F

$$\hat{\mathbf{T}}_{\mathcal{F}}=\frac{1}{2}\begin{bmatrix} U_1+U_2 & i(U_1-U_2) & \gamma_1^{-1}V_1-\gamma_2^{-1}V_2 & i\gamma_1^{-1}V_1+i\gamma_2^{-1}V_2 \\ -i(U_1-U_2) & U_1+U_2 & -i\gamma_1^{-1}V_1-i\gamma_2^{-1}V_2 & \gamma_1^{-1}V_1-\gamma_2^{-1}V_2 \\ -\gamma_1V_1+\gamma_2V_2 & -i\gamma_1V_1-i\gamma_2V_2 & U_1+U_2 & i(U_1-U_2) \\ i\gamma_1V_1+i\gamma_2V_2 & -\gamma_1V_1+\gamma_2V_2 & -i(U_1-U_2) & U_1+U_2 \end{bmatrix} \tag{61}$$

where

$$\begin{gathered} U_1=\cos(Q_1F)=\cos(N_1f), \quad V_1=\sin(Q_1F)=\sin(N_1f), \\ U_2=\cos(Q_2F)=\cos(N_2f), \quad V_2=\sin(Q_2F)=\sin(N_2f), \end{gathered} \tag{62}$$

$$N_1=\sqrt{(\varepsilon+\alpha)(\mu+\beta)}, \quad N_2=\sqrt{(\varepsilon-\alpha)(\mu-\beta)}. \tag{63}$$

To avoid confusion of symbols, in the case of the $\mathcal{F}$ layer we use the capital symbols $U_1,U_2,V_1,V_2,N_1,N_2,Q_1,Q_2$. Similarly to the $\mathcal{A}$ layers, the dependence of the transfer matrix $\hat{\mathbf{T}}_{\mathcal{F}}$ in Eq. (62) on the frequency ω and the layer thickness F comes through a single dimensionless parameter

$$f=\frac{\omega}{c}F.$$

In addition to f, there are four more independent dimensionless physical parameters in $\hat{\mathbf{T}}_{\mathcal{F}}$ which are material constants defined in Eqs. (60) and (63).

In accordance with Eqs. (18) and (19), the transfer matrices $\hat{\mathbf{T}}_{\mathcal{F}}$ of $\mathcal{F}$ layers with two opposite magnetic polarizations $\vec{M}_0$, are related by the transposition of the indices 1 and 2 in Eq. (61),

$$\begin{bmatrix}\mathcal{F}\\+\end{bmatrix}\rightleftarrows\begin{bmatrix}\mathcal{F}\\-\end{bmatrix} \quad \text{is equivalent to} 1\rightleftarrows 2. \tag{64}$$

Note in passing that depending on the specific magnetic material and the frequency range, there might be two distinctive limiting cases. For $\hat{\mu}=\hat{1}$

$$\gamma_1=N_1=\sqrt{(\varepsilon+\alpha)},\qquad \gamma_2=N_2=\sqrt{(\varepsilon-\alpha)} \tag{65}$$

and for $\hat{\varepsilon}=\overline{1}$

$$\gamma_1=N_1^{-1}=\sqrt{(\mu+\beta)^{-1}},\qquad \gamma_2=N_2^{-1}=\sqrt{(\mu-\beta)^{-1}}. \tag{66}$$

Our spectral calculations show that the ''dielectric'' limiting case (65), compared to the general case, produces a slightly different spectral asymptotic at $k\rightarrow 0$ for the lowest spectral branches with $\omega(0)=0$. Otherwise, there is no qualitative difference between the general case and the two limiting cases.

V. CHARACTERISTIC EQUATION AND SPECTRAL NONRECIPROCITY

Consider now an infinite periodic stack forming a 1D magnetic photonic crystal, and denote by L the length of its primitive cell. The Bloch eigenmodes of the crystal satisfy [27]

$$\begin{bmatrix}\vec{E}(z+L)\\ \vec{H}(z+L)\end{bmatrix}=\begin{bmatrix}\vec{E}(z)\\ \vec{H}(z)\end{bmatrix}e^{ikL}. \tag{67}$$

Using the transfer matrix (40) we recast the Bloch condition (67) as

$$[\hat{\mathbf{T}}(z+L,z)-\hat{\mathbf{I}}e^{ikL}]\begin{bmatrix}\vec{E}(z)\\ \vec{H}(z)\end{bmatrix}=0, \tag{68}$$

where

$$\hat{\mathbf{T}}(z+L,z) \tag{69}$$

is the transfer matrix of the primitive cell. Different z's correspond to different choices of the primitive cell. Notice that although the matrix $\hat{\mathbf{T}}(z+L,z)$ depends on z, its eigenvalues do not. In further consideration we set $z=0$ and place the reference point $z=0$ at one of layers' interfaces. In this case Eq. (67) reduces to

$$[\hat{\mathbf{T}}(L)-\hat{\mathbf{I}}e^{iK}]\begin{bmatrix}\vec{E}\\ \vec{H}\end{bmatrix}=0, \tag{70}$$

where K is a dimensionless form of the wave vector

$$K=kL.$$

In view of Eq. (43), the transfer matrix $\hat{\mathbf{T}}(L)$ is expressed in terms of the transfer matrices $\hat{\mathbf{T}}_m$ of the related individual layers

$$\hat{\mathbf{T}}(L)=\prod_m \hat{\mathbf{T}}_m, \tag{71}$$

where the index m runs over all layers in the primitive cell. We note that matrices $\hat{\mathbf{T}}_m$ may not commute.

According to Eqs. (67) and (70), the Bloch eigenmodes coincide with the eigenmodes of the transfer matrix $\hat{\mathbf{T}}(L)$. The corresponding eigenvalues

$$Z=e^{ikL}=e^{iK} \tag{72}$$

are the roots of the characteristic polynomial

$$\det[\hat{\mathbf{T}}(L)-Z\hat{\mathbf{I}}]=F(Z)=Z^4+P_3Z^3+P_2Z^2+P_1Z+1, \tag{73}$$

where

$$P_1=P_3^*,\quad P_2=P_2^*. \tag{74}$$

Using the substitutions

$$M(Z)=Z^{-2}F(Z),\quad Z=\cos K+i\sin K,$$

one can recast the characteristic equation

$$F(Z)=Z^4+P_3Z^3+P_2Z^2+P_1Z+1=0 \tag{75}$$

as

$$M(K)=-2+P_2+2R\cos K+2P\sin K+4(\cos K)^2=0, \tag{76}$$

where all coefficients are real

$$R=\mathrm{Re}\,P_1,\quad P=\mathrm{Im}\,P_1.$$

Equations (76) or (75) determine the electromagnetic spectrum $\omega(k)$ of the 1D magnetic photonic crystal. Indeed, in line with Eqs. (70), (71), (56), and (61), the coefficients P, R, and P_2 in Eq. (76) are functions of the matrix $\hat{\mathbf{T}}(L)$ and, thereby, functions of the frequency ω. Real solutions k_j for Eq. (76) [or, equivalently, the roots of Eq. (75) with $|Z_j|=1$] correspond to propagating Bloch waves. Complex k_j [or, equivalently, the roots of Eq. (75) with $|Z|\neq 1$] correspond to the band gaps.

If all the coefficients in Eq. (75) are real [or, equivalently, if P in Eq. (76) is zero], the corresponding spectrum $\omega(k)$ is reciprocal

$$\omega(k)=\omega(-k).$$

The appearance of complex coefficients in Eq. (73) is an indication of the spectral nonreciprocity

$$\omega(k)\neq\omega(-k).$$

An advantage of the transfer matrix approach lies in the fact that based on the set of the matrices $\hat{\mathbf{T}}_m$ for individual constitutive layers one can immediately obtain the spectrum of an arbitrarily complex stack (see, also [30,9,3]). We find the transfer matrix formalism particularly efficient in the spectral analysis of nonreciprocal magnetic stacks.

Based on the characteristic equation (76) we have computed and analyzed the electromagnetic spectra of numerous reciprocal and nonreciprocal magnetic stacks. The results are summarized in Sec. VI.

Polarization degeneracy

Up to this point the only restriction on the constitutive components of the stack we actually used was that the z axis coincides with a distinguished symmetric direction for every single layer. This restriction ensures the transverseness (34) of the plane electromagnetic waves propagating in the z direction and the applicability of the transfer matrix formalism.

An additional important specification stems from the earlier assumption that different layers are made of dielectric components (ferrites and anisotropic dielectrics), each of which can be described by generic constitutive relations (4) without exotic magnetoelectric terms (2). This implies that each of the layers, if it fills the entire space, supports perfectly the reciprocal electromagnetic spectrum $\omega(q)=\omega(-q)$. At the same time, we expect that the proper periodic array of such reciprocal components can produce strong spectral nonreciprocity $\omega(k)\neq\omega(-k)$.

In view of Eq. (53), in the absence of the linear magnetoelectric response (2), a single-layer transfer matrix $\hat{\mathbf{T}}_m$ always satisfies

$$\det\hat{\mathbf{T}}_m=1. \qquad (77)$$

The same property applies to the transfer matrix $\hat{\mathbf{T}}(L)$ of the primitive cell

$$\det\hat{\mathbf{T}}(L)=Z_1Z_2Z_3Z_4=1 \qquad (78)$$

or, equivalently,

$$k_1+k_2+k_3+k_4=0,\quad \text{where}\quad k_j=-iL^{-1}\ln(Z_j). \qquad (79)$$

In the case of a reciprocal spectrum, the relation (79) is satisfied automatically, since for every $\omega(k_j)$ there is the reciprocal solution $\omega(-k_j)$. Importantly, the relation (79) still holds in the nonreciprocal situation, when four wave vectors in (79) may have four different absolute values. Although the equality (79) may appear as a symmetry restriction, in fact it is not at all related to the magnetic symmetry G of the stack. Indeed, the restriction (79) stems solely from the equality (77) which, in turn, holds automatically if every individual layer taken separately supports a reciprocal spectrum $\omega(q)=\omega(-q)$. This is always the case, unless some of the layers display a linear magnetoelectric effect.

Consider now an important consequence of the relation (79): *regardless of whether the condition (12) is met, the spectrum $\omega(k)$ of a periodic stack must be reciprocal if it supports the polarization degeneracy*.

Indeed, let the z direction be the optical axis of the stack, so that both wave polarizations 1 and 2 for every $\vec{k}\parallel\hat{z}$ are degenerate. The polarization degeneracy implies that for a fixed frequency ω,

$$k_{1r}(\omega)=k_{2r}(\omega),\quad k_{1l}(\omega)=k_{2l}(\omega), \qquad (80)$$

where 1 and 2 denote the wave polarizations while the indices r and l stand for the right and the left directions of wave propagation, respectively. The comparison of Eq. (80) with Eq. (79) yields

$$k_{1r}(\omega)=-k_{1l}(\omega)=k_{2r}(\omega)=-k_{2l}(\omega) \qquad (81)$$

which implies the spectral reciprocity along with the twofold polarization degeneracy (80).

We can conclude that the polarization degeneracy on its own ensures the spectral reciprocity, unless a periodic stack includes components with linear magnetoelectric response. In contrast, as we show in the Appendix, the spectral nonreciprocity can coexist with polarization degeneracy in a uniform magnetoelectric medium.

VI. CONCLUSION

As we have seen in the preceding section, having the transfer matrices $\hat{\mathbf{T}}_m$ for the individual layers we can immediately obtain the explicit expression (71) for the transfer matrix $\hat{\mathbf{T}}(L)$ of a primitive cell of 1D magnetic photonic crystal. Then, having $\hat{\mathbf{T}}(L)$ and using Eqs. (70) and (76) we can find the eigenmodes and the spectrum $\omega(k)$. The transfer matrices $\hat{\mathbf{T}}_{\mathcal{A}}$ and $\hat{\mathbf{T}}_{\mathcal{F}}$ for the individual $\mathcal{A}$ and $\mathcal{F}$ layers are defined in Eqs. (56) and (61), respectively. Thus, we have everything we need to calculate the electromagnetic spectrum of an arbitrarily complex periodic array of the $\mathcal{A}$ and $\mathcal{F}$ layers.

The transfer matrix $\hat{\mathbf{T}}(L)$ from Eq. (71) depends on N_A-1 misalignment angles, where $N_{\mathcal{A}}$ is the number of $\mathcal{A}$ layers in a primitive cell. In addition to that, there are eight more independent dimensionless parameters, three of which originate from $\hat{\mathbf{T}}_{\mathcal{A}}$ (these are n_1, n_2, and $a=\omega A/c$) and the remaining five originate from $\hat{\mathbf{T}}_{\mathcal{F}}$ (these are N_1, N_2, γ_1, γ_2, and $f=\omega F/c$). Of those eight parameters, six $(n_1,n_2,N_1,N_2,\gamma_1,\gamma_2)$ are material constants relating to electric permittivity and magnetic permeability. In common materials they typically range between 0 and 10^2 or 1 and 10^2. Physically, the degree of spectral nonreciprocity can be limited by the largest of the two contributions α and β from Eq. (16) to the Faraday rotation. As soon as either of them is of the order of magnitude or greater than unity, the degree of spectral nonreciprocity can be substantial, as shown in Figs.

3 and 4. In the microwave frequency range, the dominant contribution to the Faraday rotation comes from β [33], while α is negligible. In the infrared and optical range the situation is different. By contrast, the dimensionless parameters $a=\omega A/c$ and $f=\omega F/c$ are not restricted at all, because there is no physical limitation on layer thickness. Therefore, when analyzing the electromagnetic spectra, we assign some reasonable physical values to the above six material constants, while keeping a and f arbitrary. For instance, the spectra presented in Figs. 1–4 correspond to one of the following two sets of numerical values of the material parameters:[4]

Numerical set	n_1	n_2	N_1	γ_1	N_2	γ_2
1	9.3	1.1	8.9	3.5	1.3	23
2	19	1.1	8.9	3.5	1.3	23

(82)

We have analyzed the electromagnetic spectra of numerous periodic magnetic stacks with three and more layers in a primitive cell. Based on the numerical results, we have come to the conclusion that the general picture of the electromagnetic spectrum appears not to be particularly sensitive to a specific composition of the primitive cell. For instance, the spectrum does not qualitatively change if we switch from three-layered to four- or six-layered periodic arrays, provided that the criterion (14) for the spectral nonreciprocity holds. For a given set of property tensors $\hat{\varepsilon}_{\mathcal{A}}$ from Eq. (15) and $\hat{\varepsilon}_{\mathcal{F}}$ and $\hat{\mu}_{\mathcal{F}}$ from Eq. (16), it is always possible to pick the optimal layer thicknesses A and F and misalignment angles φ to achieve the desired spectral features. Since there is no apparent qualitative difference between nonreciprocal spectra in stacks with different numbers of layers in a primitive cell, we consider the simplest case of the three-layered periodic array (23) in Fig. 6 in more detail, keeping in mind that all the spectral features described below are common for all nonreciprocal periodic stacks.

In the case when the misalignment angle φ approaches 0 or $\pi/2$, as well as in the case when one of the thicknesses A or F vanishes, the electromagnetic spectrum degenerates into a perfectly reciprocal one, as shown in Figs. 1 and 2 (for explanations, see Sec. III B 1). Otherwise the spectrum displays quite appreciable nonreciprocity, as shown in Figs. 3 and 4. This fact confirms the important conclusion that magnetic photonic crystals develop strong spectral nonreciprocity $\omega(\vec{k})\neq\omega(-\vec{k})$ even when each constitutive component supports perfectly the reciprocal electromagnetic spectrum $\omega(\vec{q})=\omega(-\vec{q})$. As we mentioned before, all known uniform dielectric materials, both magnetic and nonmagnetic, are either perfectly reciprocal (the overwhelming majority), or support extremely small spectral nonreciprocity (the magnetoelectrics). Thus, magnetic photonic crystals can develop some remarkable spectral features (e.g., strong spectral nonreciprocity) that can neither be found in uniform magnetic and nonmagnetic media nor in nonmagnetic composite media.

[4]In real situations, all material parameters, especially the gyrotropic ones, may depend on the frequency ω. This will not qualitatively change the general spectral picture of a magnetic photonic crystal.

A. The frozen mode

In the Introduction we briefly discussed the possibility of one-way transparency of magnetic photonic crystals [see Eq. (9) and comments thereafter]. It is particularly remarkable, that being transparent for a certain plane wave, a magnetic photonic crystal can trap the radiation propagating in the opposite direction in the form of a coherent plane wave with $\partial\omega/\partial k=0$ and $\partial\omega^2/\partial k^2=0$ (here $k=k_z$). Clearly, such a situation is unique to magnetic photonic crystals with strong spectral nonreciprocity. If the frequency Ω relates to just a single pair of the wave vectors $[k_1,k_2]$ (as shown in Fig. 5), the magnetic stack displays a one-way transparency for this particular frequency Ω and $\vec{k}\| z$. In the course of a numerical analysis of electromagnetic spectra we tried to understand how realistic it is to arrange a magnetic periodic stack with the spectrum $\omega(k)$ displaying such a property. We have found that for a given set of property tensors $\hat{\varepsilon}$ and $\hat{\mu}$ from Eqs. (15) and (16), it is always possible to choose the layer thicknesses A and F and/or the misalignment angle φ so that at some frequency Ω one of the spectral branches develops a stationary inflection point (9). In both numerical examples presented in Figs. 3 and 4, one can find the corresponding characteristic frequency Ω on one of the spectral branches; the respective stationary inflection points are designated with a circle. In either case the frequency Ω relates to just a single pair $[k_1,k_2]$ of the wave vectors, one of which corresponds to a frozen wave. We think that the case of one-way transparency with a frozen backward wave, as well as its possible ramifications, warrants further investigation, which will be done elsewhere.

B. The role of the static magnetic field $\vec{H}_0$. Tunability

The role of the static magnetic field $\vec{H}_0$ is particularly important when the ferromagnetic $\mathcal{F}$ layers are made of magnetically soft material. In such a case the external magnetic field $\vec{H}_0\| z$ aligns the magnetizations $\vec{M}_0$ of the individual $\mathcal{F}$ layers with the z direction. By changing $\vec{H}_0$ one can substantially alter the electromagnetic spectrum of a magnetic photonic crystal. Let us consider several possibilities.

In line with Eqs. (20), (18), and (19), switching the direction of the external field $\vec{H}_0\| z$ to the opposite direction causes the following transformation:

$$\vec{M}_0\rightarrow-\vec{M}_0,\quad \omega(\vec{k})\rightarrow\omega(-\vec{k}).$$

In the case (9) of the frozen backward mode, this provides the way to easily reverse the direction of one-way transparency of a magnetic stack.

Consider now a particular case

$$\begin{bmatrix}\mathcal{A}\\0\end{bmatrix}\begin{bmatrix}\mathcal{A}\\\pi/2\end{bmatrix}\begin{bmatrix}\mathcal{F}\\+\end{bmatrix}$$

of the three-layered periodic array (23) in Fig. 6. If $\vec{H}_0 \| z$, the magnetic symmetry (29) of this array ensures spectral reciprocity $\omega(k)=\omega(-k)$ for $\vec{k}\| z$, as shown in Fig. 2. The application of the criterion (14) shows that the deviation of the static magnetic field $\vec{H}_0$ from the z direction will cause the violation of spectral reciprocity, unless $\vec{H}_0$ remains confined to the *xz* or *yz* plane. So, here we can turn the spectral nonreciprocity on and off by changing the direction of the external magnetic field. Another effect of magnetic field deviation will be the loss of the transverseness (34) of plane electromagnetic waves with $\vec{k}\| z$.

Finally, as we have already mentioned, the material tensor $\hat{\mu}_{\mathcal{F}}$ in Eq. (16) strongly depends on $\vec{H}_0$. Therefore, by changing the magnitude of $\vec{H}_0 \| z$ one can effectively control the dispersion relations. In the case of reciprocal magnetic photonic crystals, this question has been addressed, for instance, in [7], [8], [34], and [32].

C. Multidimensional photonic crystals

Let us now make a few comments on 2D and 3D magnetic photonic crystals. All considerations of Sec. II, including the symmetry conditions for spectral reciprocity and nonreciprocity, are equally applicable to the case of multidimensional magnetic photonic crystals. Generally, when dealing with given constitutive components (e.g., nonmagnetic dielectrics and ferrites), it is much easier to arrange a 2D or 3D magnetic photonic crystal, which would comply with the symmetry condition (14) for spectral nonreciprocity, than it is in the case of 1D periodic arrays. For instance, in two or three dimensions, the presence of dielectric anisotropy δ in Eq. (15) may not be a requirement any more. But, when it comes to the spectral calculations, the difficulties emerging in multidimensional magnetic photonic crystals are much greater. In addition to well-known problems with nonmagnetic structures (see, for instance, [1]), the case of multidimensional magnetic photonic crystals can bring another serious problem involving the magnetic field uniformity. This problem is caused by the inevitable presence of relatively strong inhomogeneous demagnetization fields within the photonic crystal. For more details, see [32] and references therein.

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APPENDIX

1. The transfer matrix for magnetoelectric media

In dielectric materials with linear magnetoelectric response the generic constitutive relations (4) should be replaced with more general ones

$$\vec{D}=\hat{\varepsilon}\vec{E}+\hat{\chi}\vec{H}, \quad \vec{B}=\hat{\mu}\vec{H}+\hat{\chi}^T\vec{E}, \tag{A1}$$

where the asymmetric tensor $\hat{\chi}$ describes the linear magnetoelectric response (2).

In line with our initial assumption (34), let the direction z of wave propagation coincide with one of the symmetry axes of the magnetoelectric medium. According to Eqs. (35) and (A1), the monochromatic plane waves

$$\vec{E}(z)=\vec{E}e^{iqz}, \quad \vec{H}(z)=\vec{H}e^{iqz}$$

in a uniform magnetoelectric medium are determined by the system of four linear equations,

$$(n\hat{\sigma}-\hat{\chi}^T)\vec{E}-\hat{\mu}\vec{H}=0, \quad \hat{\varepsilon}\vec{E}+(n\hat{\sigma}+\hat{\chi})\vec{H}=0, \quad n=\frac{cq}{\omega}. \tag{A2}$$

Here

$$\vec{E}=\begin{bmatrix} E_x \\ E_y \end{bmatrix}, \quad \vec{H}=\begin{bmatrix} H_x \\ H_y \end{bmatrix}.$$

The system (A2) is a generalization of Eq. (45). The corresponding characteristic equation

$$\det[(n\hat{\sigma}-\hat{\chi}^T)+\hat{\mu}(n\hat{\sigma}+\hat{\chi})^{-1}\hat{\varepsilon}]=0 \tag{A3}$$

may have, instead of Eq. (47), four solutions n_1, n_2, n_3, and n_4 with different absolute values. The corresponding electromagnetic spectrum $\omega(q)$ does not reduce to two pairs of branches (46) with equal frequencies ω and equal and opposite wave vectors ($\pm q_1$ and $\pm q_2$). In other words, such a medium may display the property $\omega(q)\neq\omega(-q)$ of spectral nonreciprocity.

2. Examples

a. Orthogonal magnetic symmetry

Let us consider an antiferromagnetic crystal with orthorhombic magnetic symmetry

$$G\equiv mmm', \tag{A4}$$

which is compatible with the criterion (14) of spectral nonreciprocity for $\vec{q}\| z$. Given the symmetry group (A4) of the crystal, the property tensors $\hat{\varepsilon}$, $\hat{\mu}$, and $\hat{\chi}$ can be represented in the block-diagonal form [14]

$$\hat{\varepsilon}=\begin{bmatrix} \varepsilon_{x,x} & 0 & 0 \\ 0 & \varepsilon_{y,y} & 0 \\ 0 & 0 & \varepsilon_{z,z} \end{bmatrix}, \quad \hat{\mu}=\begin{bmatrix} \mu_{x,x} & 0 & 0 \\ 0 & \mu_{y,y} & 0 \\ 0 & 0 & \mu_{z,z} \end{bmatrix},$$

$$\hat{\chi}=\begin{bmatrix} 0 & \chi_{x,y} & 0 \\ \chi_{y,x} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A5}$$

The tensor $\hat{\chi}$ of the linear magnetoelectric response is real and asymmetric

$$\chi_{y,x} \neq \chi_{x,y}.$$

The substitution of (A5) into (A2) yields the following solutions for two possible polarizations 1 and 2:

$$\begin{bmatrix} \vec{E}_1 \\ \vec{H}_1 \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ \dfrac{n-\chi_{x,y}}{\mu_{y,y}} \end{bmatrix}, \quad \begin{bmatrix} \vec{E}_2 \\ \vec{H}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \\ -\dfrac{n+\chi_{y,x}}{\mu_{x,x}} \\ 0 \end{bmatrix}. \tag{A6}$$

As to the refractive index $n=qc/\omega$, it takes on one of the following four values [35]:

$$\begin{aligned} n_{1r}&=\sqrt{\varepsilon_{x,x}\mu_{x,x}}+\chi_{x,y}, \\ n_{1l}&=-\sqrt{\varepsilon_{x,x}\mu_{x,x}}+\chi_{x,y}, \quad \text{polarization 1}, \\ n_{2r}&=\sqrt{\varepsilon_{y,y}\mu_{y,y}}-\chi_{y,x}, \\ n_{2l}&=-\sqrt{\varepsilon_{y,y}\mu_{y,y}}-\chi_{y,x}, \quad \text{polarization 2}, \end{aligned} \tag{A7}$$

where the indices r and l stand for the right ($q>0$) and the left ($q<0$) directions of wave propagation, respectively. The explicit expressions for all four eigenmodes are

$$\begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix}=e^{iq_{1r}z}\begin{bmatrix} 1 \\ 0 \\ 0 \\ \sqrt{\dfrac{\varepsilon_{x,x}}{\mu_{y,y}}} \end{bmatrix},$$

$$\begin{bmatrix} \vec{E}_1(z) \\ \vec{H}_1(z) \end{bmatrix}=e^{iq_{1l}z}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -\sqrt{\dfrac{\varepsilon_{x,x}}{\mu_{y,y}}} \end{bmatrix},$$

$$\begin{bmatrix} \vec{E}_2(z) \\ \vec{H}_2(z) \end{bmatrix}=e^{iq_{2r}z}\begin{bmatrix} 0 \\ 1 \\ -\sqrt{\dfrac{\varepsilon_{y,y}}{\mu_{x,x}}} \\ 0 \end{bmatrix}, \tag{A8}$$

$$\begin{bmatrix} \vec{E}_2(z) \\ \vec{H}_2(z) \end{bmatrix}=e^{iq_{2l}z}\begin{bmatrix} 0 \\ 1 \\ \sqrt{\dfrac{\varepsilon_{y,y}}{\mu_{x,x}}} \\ 0 \end{bmatrix}.$$

All four wave vectors $q=n\omega/c$ in Eq. (A7) have different absolute values.

In the limiting case of zero magnetoelectric response when $\hat{\chi}\to 0$, we have instead of Eq. (A7) the following:

$$n_{1r}=-n_{1l}=n_1=\sqrt{\varepsilon_{x,x}\mu_{x,x}}, \quad n_{2r}=-n_{2l}=n_2=\sqrt{\varepsilon_{y,y}\mu_{y,y}},$$

$$q_{1r}=-q_{1l}=q_1, \quad q_{2r}=-q_{2l}=q_2,$$

which is consistent with Eqs. (46) and (47).

Let us find the transfer matrix $\hat{\mathbf{T}}(z)$ in a uniform magnetoelectric medium with constitutive relations (A5). Using Eqs. (50) and (A6) we get

$$\hat{\mathbf{T}}(z)=\hat{\mathbf{W}}(z)\hat{\mathbf{W}}^{-1}(0),$$

where

$$\hat{\mathbf{W}}(z)=\begin{bmatrix} e^{iq_{1r}z} & e^{iq_{1l}z} & 0 & 0 \\ 0 & 0 & e^{iq_{2r}z} & e^{iq_{2l}z} \\ 0 & 0 & -\sqrt{\dfrac{\varepsilon_{y,y}}{\mu_{x,x}}}e^{iq_{2r}z} & \sqrt{\dfrac{\varepsilon_{y,y}}{\mu_{x,x}}}e^{iq_{2l}z} \\ \sqrt{\dfrac{\varepsilon_{x,x}}{\mu_{y,y}}}e^{iq_{1r}z} & -\sqrt{\dfrac{\varepsilon_{x,x}}{\mu_{y,y}}}e^{iq_{1l}z} & 0 & 0 \end{bmatrix}.$$

Notice that the determinant of $\hat{\mathbf{T}}(z)$ is not unity anymore but

$$\det \hat{\mathbf{T}}(z)=\exp\left[2i\frac{\omega}{c}(\chi_{x,y}-\chi_{y,x})z\right]\neq 1. \tag{A9}$$

b. Tetragonal magnetic symmetry

The above expressions (A5) through (A9) can be easily rewritten for the case of tetragonal magnetic symmetry $4/m'mm$. Indeed, according to [14], the material tensors $\hat{\varepsilon}$,

$\hat{\mu}$, and $\hat{\chi}$ for the magnetic symmetry of $4/m'mm$ can be obtained from those for the symmetry group mmm' by simple substitution

$$\varepsilon_{x,x}=\varepsilon_{y,y}\equiv\varepsilon,\quad \mu_{x,x}=\mu_{y,y}\equiv\mu,\quad \chi_{x,y}=-\chi_{y,x}\equiv\chi \tag{A10}$$

in (A5) through (A9). In particular, we have instead of (A7)

$$n_{1r}=n_{2r}=\sqrt{\varepsilon\mu}+\chi,\quad n_{1l}=n_{2l}=-\sqrt{\varepsilon\mu}+\chi, \tag{A11}$$

which describes a nonreciprocal spectrum with polarization degeneracy. By contrast (see Sec. V A), polarization degeneracy in periodic magnetic stacks would ensure the spectral reciprocity.

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