

ω -sequences in the uB-power set

In a recent paper, Müller and Sargsyan introduce the uB-powerset, \mathcal{A}^∞ , as a natural candidate for a higher order analogue to Γ^∞ , the set of all universally Baire sets. They prove that, if κ is supercompact and there is a proper class of inaccessible limits of Woodins, then in any forcing extension that collapses 2^{2^κ} to \aleph_0 , $L(\mathcal{A}^\infty)$ is a model of AD^+ . In fact, they show that, under the mentioned large cardinal assumption, Weak Sealing holds for $L(\mathcal{A}^\infty)$, in any $\text{Col}(\omega, 2^{2^\kappa})$ -extension. In continuation of this previous work, Müller, Sargsyan and I are aiming to show that, in any $\text{Col}(\omega, 2^{2^\kappa})$ -extension, $L((\mathcal{A}^\infty)^\omega)$ is a model of AD^+ . As a first step towards this goal, we will establish that the set of ordinals coded by elements of \mathcal{A}^∞ has cofinality ω , thereby also showing that $L((\mathcal{A}^\infty)^\omega)$ properly extends $L(\mathcal{A}^\infty)$. In addition to distinguishing two natural models of “ AD^+ and $V \neq L(\wp(\mathbb{R}))$ ”, this work will negatively answer the question whether \mathcal{A}^∞ is the maximal $C \subseteq \wp(\Gamma^\infty)$ such that $L(C) \models \text{AD}^+$.