2016-07-26 9:30 JOHN STEEL

- Defining $\mathcal{S}_{\gamma,\ell} = \mathcal{S}$ on (M, κ, α_0)
- Have $\mathcal{U}_{\gamma,\ell}$ on M
- Comparisons $M^{\mathbb{C}}_{\gamma,\ell}$. Have $M^{\mathcal{S}}_{\gamma}$. Fix γ, ℓ. $\mathcal{S} = \mathcal{S}_{\gamma,\ell}$.

$$\begin{array}{ccc} (M, \kappa, \alpha_0) & \xrightarrow{id, \pi} & M \\ \mathcal{S} & & \mathcal{T} \\ & & \mathcal{U} \end{array}$$

$E^{\mathcal{S}}_{\gamma}$ = least disagreement with $M_{\gamma,\ell}$

Let α = the least s.t. $\mathrm{crit}(E^{\mathcal{S}}_{\gamma}) < \lambda^{\mathcal{S}}_{\alpha}$.

- $\underline{\alpha \text{ stable}}$: $M^{\mathcal{S}}_{\gamma+1} = \mathrm{Ult}(P, E^{\mathcal{S}}_{\gamma})$, $P \trianglelefteq M^{\mathcal{S}}_{\alpha}$ longest possible
  $\lambda^{\mathcal{S}}_{\gamma+1} = \lambda(E^{\mathcal{S}}_{\gamma})$, $\pi_{\gamma+1} = M^{\mathcal{S}}_{\gamma+1} \to M^{\mathcal{T}}_{\gamma+1}$
  by shift lemma

in fact $M^{\mathcal{S}}_{\gamma+1} = \mathrm{Ult}(P, E^{\mathcal{S}}_{\gamma})$ in any case

θ unstable: $\lambda_{\theta} \leq \alpha_{\theta}$. Then $P = M^{\mathcal{S}}_{\theta}$

(a) If $M^{\mathcal{S}}_{\gamma+1}$ not a model of $\mathcal{U}$ then $\gamma+1$ is stable.
(b) If $M^{\mathcal{S}}_{\gamma+1} = M^{\mathcal{U}}_{\tau}$ some τ. (S. $s^{\mathcal{S}}_{\gamma+1} = s^{\mathcal{U}}_{\tau}$)
then we declare $\gamma+1$ unstable

$$M^{\mathcal{S}}_{\gamma+2} = \mathrm{Hull}^{M^{\mathcal{S}}_{\gamma+1}}_{k+1}(\alpha_{\gamma+1} \cup i_{\alpha,\gamma+1}(q))$$

$$\alpha_{\gamma+1} = \sup i_{0,\gamma+1}[\alpha_0]$$

Now look at $E^{\psi}_{\gamma+1}$

$$\lambda^{\psi}_{\gamma+1} = \inf(\alpha_{\gamma+1}, \lambda(E^{\psi}_{\gamma+1}))$$
$$\lambda(E^{\psi}_{\gamma+1}) = \lambda^{\psi}_{\gamma+2}$$

Limit case: θ limit: Get $[0,\theta]_s$ M^{ψ}_{θ} by looking at $\mathcal{T}$ and π_θ.

If some $\alpha <_s \theta$ is stable we declare θ stable. In fact θ is unstable iff M^{ψ}_{θ} is not on $\mathcal{U}$ $s^{\psi}_{\theta} \neq s^{\psi}_{\tau}$ any τ.

If θ is unstable, create $M^{\psi}_{\theta+1}$ as above.

Now adapt the proof that (M, Ψ) iterates to an $(M^c_{\nu_0,k_0}, \Omega^c_{\nu_0,k_0})$ to the (M, K, α_0)-side

Key things: Have the strategy Ψ for M and $\Psi^{(id,\pi)} = \Lambda$ for (M, K, α_0)

For any R (e.g. some $M^c_{\nu,e}$) can define (R, Ψ, Ω) coiteration of M, (M, K, a_0) with trees $(\mathcal{U}, \mathcal{Y})$ (For $R = M_{\nu,e}$ it is $\mathcal{U}_{\nu,e}, \mathcal{Y}_{\nu,e}$)

If we have $j: V \to M$ then
$j(\mathcal{U}, \mathcal{Y})$ = the $j(R; \Psi, \Lambda)$ coiteration of M vs (M, K, α_0)

The strategies on right are the same by UB

So if $j(R)|\langle\eta,i\rangle = R|\langle\eta,i\rangle$ then $(\mathcal{U},\mathcal{Y})$ and $j(\mathcal{U},\mathcal{Y})$ agree until looking at $j(R)|\langle\eta,i\rangle$.

Condensation lemma works similarly.

To compute HOD we will use UBH in M where (M,Ω) is a lbr hod pair. Appropriate form of UBH holds in $L[E]$-models.

Assuming AD^+ and (M,Ω) is an lbr hod pair with scope HC. Let η be a cutpoint of M and let g be M-generic / $\mathrm{col}(\omega,\eta)$. Suppose

$M[g] \models \mathcal{T}$ is a plus-two IT on ~~M~~ me $\models$ extenders on the sequence with cr. pt. $> \eta$.

Then $M[g] \models \mathcal{T}$ has at most one cof. w.f. branch

<u>Plus-two means</u>: Have $\rho_\alpha^{\mathcal{T}}$ with

$$(\rho_\alpha^{\mathcal{T}})^{++M_\alpha^{\mathcal{T}}} < \lambda(E_\alpha^{\mathcal{T}}) < (\rho_\alpha^{\mathcal{T}})^{+++M_\alpha^{\mathcal{T}}}$$

and we also require that $\mathcal{T}$ is non-dropping.

The proof is like the proof of UBH in $L[E]$-models.

<u>Sketch</u> If not, by taking hulls get

$N = \mathrm{Hull}_\omega^{M|\zeta}(\eta)$ where ζ large enough s.t. $M|\zeta \models$ Thm is false

$N[h]$ has a counterexample $\mathcal{T}$ where h on $\mathrm{col}(\omega,\bar{\eta})$

$\sigma^\frown b$, $\sigma^\frown c$ for instance

$\Phi(\sigma^\frown b)$, $\Phi(\sigma^\frown c)$ Can be realized in M.

Here we need the $+2$ - part.

Using pointwise definability for N, can compare the phalanxes $\Phi(\sigma^\frown b)$, $\Phi(\sigma^\frown c)$, but this gives contradiction.

Now we get generic interpretability of the strategies

see Grigor's Thesis

<u>Gen. Int. Lemma</u> (AD^+). Let (P, Σ) be an lbr hod pair with scope HC s.t. $Code(\Sigma)$ is Suslin-co-Suslin. Let

$$P \models ZFC^- + \delta \text{ is Woodin}$$

Then there is a term $\tau \in P$ s.t. whenever $i : P \longrightarrow Q$ is an iteration map associated to a non-dropping stack s on P by Σ and g is generic for $Col(\omega, <i(\delta))$ over Q then $i(\tau)^g = \Sigma_{s, <i(\delta)} \restriction HC^{Q[g]}$

$\uparrow$

$Q|\langle i(\delta), 0\rangle$

<u>Pf</u> (Sketch) For $\xi < \eta < \delta$ and $k < \omega$ define a term $\tau_{\xi,k,\eta}$ s.t. whenever (we focus on the case $P = Q$; $i = id$) g is $\mathbb{P}$-generic / $Col(\omega, \eta)$ then $\cancel{\tau^g = \Sigma_{N|\langle\delta,0\rangle}}$

$$\tau^g = \Sigma_{\langle \xi, k\rangle} \restriction HC^{P[g]}$$

Do the $L[E]$ - construction in P up to δ above η with $+2$ - extenders.

$P|\langle\xi,k\rangle$ iterates onto a model on the construction via a tree $\mathcal{T}$; $\pi : P|\langle\xi,k\rangle \xrightarrow{\mathcal{T}} M^{\mathbb{C}}_{\nu,k}$. Then $\Sigma_{\langle\xi,k\rangle}$ is the pullback strategy, i.e.

$$\Sigma_{\langle\xi,k\rangle} = (\Omega^{\mathbb{C}}_{\nu,k})^{\pi} \quad \text{(this holds in } P\text{)}$$

Given $\mathcal{U}$ ctble in $P[g]$ on $P|\langle\xi,k\rangle$: $\pi\mathcal{U}$ is on $M_{\nu,k}$ and $\pi\mathcal{U}^*$ comes from lifting, i.e.

$$\text{lift}(\pi(\mathcal{U}), M^{\mathbb{C}}_{\nu,k}, \mathbb{C}) = \langle \pi(\mathcal{U})^*, \ldots \rangle$$

Now we assume UBH, and this applies to $\pi\mathcal{U}^*$. But then $\pi\mathcal{U}^*$ is continuously ill founded off its unique branch.