Math 232b: Algebraic Number Theory

Winter 2018 Course Information and Syllabus
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Lectures: M,W,F 12:00 - 12:50 in Rowland Hall 306.
Office Hours: W 11:00 - 12:00, RH 540c.
Also, please feel free to email me to set up an appointment.

Course Overview

Math 232b is the second quarter of a year-long introduction to algebraic number theory.

In Math 232a we developed a vocabulary for discussing the arithmetic of algebraic number fields. We introduced Dedekind domains, focusing on the ring of integers $\mathcal{O}_K$ of a number field K. We proved that such a ring has unique factorization of ideals into products of prime ideals, and defined the ideal class group $\mathrm{Cl}(\mathcal{O}_K)$. We proved that $\mathrm{Cl}(\mathcal{O}_K)$ is a finite abelian group that tells us about factorizations of elements of $\mathcal{O}_K$. We used ideas from the geometry of numbers to prove Minkowski's bound, which helps us compute these groups. We also used these ideas to prove Dirichlet's theorem on the structure of the group of units in $\mathcal{O}_K$, and to prove Hermite's theorem that there are finitely many number fields K with $|\operatorname{disc}(K)| \leq X$.

One of the main goals of algebraic number theory is to understand how the arithmetic of number fields allows us to classify solutions to certain Diophantine equations. For example, we saw how studying quadratic fields could tell us which primes p can be written as $p = x^2 + y^2$ or as $p = x^2 + xy + y^2$, and we saw how understanding the arithmetic of cyclotomic fields could help us solve certain cases of Fermat's Last Theorem. We will continue these investigations in Math 232b, emphasizing examples and computations as we go.

Major Topics

1. Localization.
2. Relative Extensions of Number Fields– Galois Theory Applied to Prime Decomposition.
3. Local Fields.
4. The Kronecker-Weber Theorem.
5. Introduction to Analytic Number Theory.
(Chebotarev Density Theorem, Analytic Class Number Formula, etc.)
6. Introduction to Class Field Theory.
7. Binary Quadratic Forms and Class Groups.

We will definitely cover the first few topics, but will not cover all the last few topics in detail.

Course Texts

1. *Number Fields* by Daniel Marcus.
2. *Number Rings*: Course notes by Peter Stevenhagen.
   Available online: `http://websites.math.leidenuniv.nl/algebra/ant.pdf`.
3. *Algebraic Number Theory*: Course notes by Matt Baker.
   Available online: `http://people.math.gatech.edu/~mbaker/pdf/ANTBook.pdf`.
4. *Algebraic Number Theory*: Course notes by Andrew Sutherland.
   Available online: `http://math.mit.edu/classes/18.785/2017fa/lectures.html`.

The first part of the course will follow Chapters 4 and 5 of Baker's notes pretty closely. We will also fill in some material from Stevenhagen's notes that we skipped in Math 232A. Sutherland's notes will be a good reference for the later part of the course.

Prerequisites

If you would like to take this course but did not take Math 232A, please email me to set up a meeting.

The main prerequisite is a quarter of Algebraic Number Theory covering the basics of number fields, roughly Chapters 1-3 of Baker's notes. Students should have taken a good graduate course in algebra on the level of the Math 230 sequence. Math 232B will use more algebra than Math 232A did. The unit on analytic number theory will require some background in real and complex analysis on the level of an introductory graduate course.

Grading

- Homework: 60%
- Final Paper (Select a topic by March 2nd. Due Monday, March 19th): 30%
- Final Presentation (During the week of March 19th): 10%

Homework in Math 232B will be similar to the homework from Math 232A. Doing these homework problems will be the best way to absorb the material from the course. I encourage you to work together on these problem sets. You can use any outside resources (other textbooks, papers, the internet, etc.) but if you find a solution somewhere you must acknowledge it.

We will not have any exams or a final problem set. Instead, we will have a Final Paper and Final Presentation. There are many interesting topics related to this course that we will not have time to cover in detail. The idea of the final paper is that you will select one of these and write up an 8-10 page expository paper on your topic. I will update a list of possible paper topics throughout the course, but you can also feel free to find your own topic. During Finals Week each student will give a short (approximately 20 minutes) presentation on their final paper topic.