# Math 232C: Algebraic Number Theory

**Winter 2018 Course Information and Syllabus**

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**Lectures**: M,W,F 12:00 - 12:50 in Rowland Hall 306.
**Office Hours**: W 11:00 - 12:00, Rowland Hall 540c.
Also, please feel free to email me to set up an appointment.

## Course Overview

Math 232C is the third quarter of a year-long introduction to algebraic number theory.

In Math 232A we developed a vocabulary for discussing the arithmetic of algebraic number fields, and we continued in this direction during Math 232B. We began the last quarter by talking about localization. We then spent a few weeks discussing the interactions between Galois theory and prime decomposition. We stated the Chebotarev density theorem and saw some applications. We gave an overview of some analytic number theory, including the Prime Number Theorem and Dirichlet's Theorem for Primes in Progressions. We spent several weeks covering the basics of absolute values on fields and focused specifically on the p-adic absolute value and the field of p-adic numbers. In the final part of the course we talked about extensions of absolute values, extensions of local fields, and the relationship between local and global extensions.

We will begin Math 232C by applying some of what we saw last quarter to sketch a proof of the Kronecker-Weber Theorem. We will then spend several weeks giving a survey of the highlights of class field theory. We will not give many proofs (this would take much more time than we have), but hope to give an overall feel for the subject.

After finishing our tour of Class Field Theory we will discuss quadratic forms in more depth. We will talk about quadratic forms over $\mathbb{Q}$, over $\mathbb{Z}$, and over $\mathbb{Q}_p$. Our goal is to work towards a proof of the Hasse-Minkowski Theorem, which is an example of a *Local-Global Principle*.

Instead of just asking whether or not an integer is represented by a quadratic form, we can ask enumerative questions about how many times an integer is represented. These questions lead naturally to the theory of *Theta Functions*. In the final part of the course we will use Sums of Squares formulas to give a motivated introduction to *Modular Forms*.

## Major Topics

1. The Kronecker-Weber Theorem.
2. Introduction to Class Field Theory.
3. Binary Quadratic Forms and Ideal Class Groups.
4. Quadratic Forms and the Hasse-Minkowski Theorem.
5. Introduction to Modular Forms.

# Course Texts

1. *Algebraic Number Theory*: Course notes by Andrew Sutherland.
   Available online: `http://math.mit.edu/classes/18.785/2017fa/lectures.html`.

2. *A Course in Arithmetic* by Jean-Pierre Serre.
   `http://www.springer.com/us/book/9780387900407`

The first part of the course will follow parts of the last few lectures of Sutherland's course notes pretty closely. Sutherland's notes cover much more material than is reasonable to cover in lecture, so we will not go over everything in detail.

The second part of the course will cover parts of Serre's *A Course in Arithmetic*. In particular, we plan to study quadratic forms in some detail, leading up to a proof of the Hasse-Minkowski Theorem. We will skip over the proof of Dirichlet's Theorem, but will end the course with an introduction to modular forms.

## Prerequisites

If you would like to take this course but did not take Math 232B, please email me to set up a meeting.

The main prerequisite for the first part of the course is a good graduate course in Algebraic Number Theory covering the basics of number fields and local fields. If you are comfortable with the material of the Algebraic Number Theory notes of Milne or Baker, that will be enough. Students should have taken a good graduate course in algebra on the level of the Math 230 sequence.

The material from the second part of the course (*A Course in Arithmetic*) is separate from the material of the first part of the course and has fewer prerequisites.

## Grading

There will be a final paper for Math 232C similar to the final paper for Math 232B. The paper should be an expository 8-10 page summary of a topic related to the material of this course.

The paper will be due on **Monday, June 11th at 3:00 PM**.
Please email me with a paragraph describing your paper topic by **Tuesday, May 29th**.
We will have optional final presentations at the end of the course. I really enjoyed the presentations we had for Math 232B and think this is an excellent opportunity for you to work on your mathematical presentation skills.

There will not be any required homework for Math 232C. Students who hand in a final paper that demonstrates sincere effort will receive an A for the course.