Math 233a: Algebraic Geometry

Fall 2016 Course Information and Syllabus

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Lectures: M,W,F 12:00 - 12:50 in Rowland Hall 440R.
Office Hours: Wednesday 10:30 - 11:30, RH 540c.
Also, please feel free to email me to set up an appointment.

Course Overview

Algebraic geometry is a central subject in mathematics that has close connections to number theory, differential geometry, representation theory, and many other areas. The overall goal is to understand algebraic varieties, zero sets of collections of polynomial equations. We will introduce much of the basic theory of affine and projective varieties, and work to develop a rigorous theory of maps between varieties, dimension, and singularities. We will introduce a large set of examples and emphasize explicit computations.

Topics

Affine varieties, the Zariski topology, regular functions and coordinate rings, projective varieties, morphisms between varieties, rational maps, products of varieties, Examples: Veronese, Segre, Grassmannians, conics and quadrics, rational curves, dimension, degree, singularities and tangent spaces.

This list above is more than we can cover in detail in 10 weeks, but we will at least provide the background necessary to investigate these topics in more detail in future quarters.

Course Texts

The primary textbook will be *Basic Algebraic Geometry I*, by Shafarevich. It is available to UCI students for free through SpringerLink. This is a very comprehensive book and should be a really useful reference for the rest of the 233 sequence.

Additional References

- Smith, Kahanpää, Kekäläinen, Traves, *An Invitation to Algebraic Geometry.* This is a short book that is written as an introduction to the main ideas of algebraic geometry at a very accessible level. I highly recommend it.
- Gathmann's course notes have been used for the 233 sequence in the past: `http://www.mathematik.uni-kl.de/~gathmann/class/alggeom-2014/main.pdf`.

- Hassett, *Introduction to Algebraic Geometry.* This is another very accessible introduction to algebraic geometry that has an emphasis on explicit computations and algorithmic problems.
- Harris, *Algebraic Geometry: A First Course.* This book is a great place to see lots of great examples. It does not have as formal a theorem-proof style as some of the other texts, so it is good to read this together with a book like Shafarevich.
- One idea about how to learn algebraic geometry is to work one dimension at a time, starting with algebraic curves, then surfaces, and then higher dimensions. Fulton has freely available notes on Algebraic Curves that serve as a great introduction to algebraic geometry overall: `http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf`.
- Reid, *Undergraduate Algebraic Geometry.* A very readable introduction to the subject, emphasizing examples from classical algebraic geometry. Now freely available online: `https://homepages.warwick.ac.uk/staff/Miles.Reid/MA4A5/UAG.pdf`.

There are two additional texts that are standard references for graduate algebraic geometry courses: Vakil's *The Rising Sea: Foundations of Algebraic Geometry* and Hartshorne's *Algebraic Geometry.* Both pursue a more abstract approach than I will in the first quarter, emphasizing sheaves and schemes. Still, they are useful to keep in mind for further study.

Prerequisites

Students should be comfortable with algebra on the level of the 230 sequence. There will be times when we need statements from commutative algebra- I will likely not prove these in lecture, but will give you references where you can see detailed proofs. I think that learning commutative algebra and algebraic geometry together is a good idea and would recommend reading through parts of Atiyah and MacDonald's *Commutative Algebra* while going through this course.

Grading

- Weekly Homework: 60%
- In-Class Exam (Friday, October 28): 20%
- Final Problem Set (Due Monday, December 5th): 20%

I will give weekly homework assignments that will be a very important part of the course. I feel that algebraic geometry is a subject where you really need to do problems to absorb the key definitions and ideas. I encourage you to work together on these problem sets. We will also have one in-class exam. It is designed not to be difficult, but just to check that you understand the main definitions and can work with basic examples. We will have a final problem set instead of a final exam. It will be like a regular problem set, but longer, and it should be done individually.