Math 354: Number Theory

Spring 2015 Course Information and Syllabus

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Lectures: Tu,Th 2:30 - 3:45 in LOM 215.

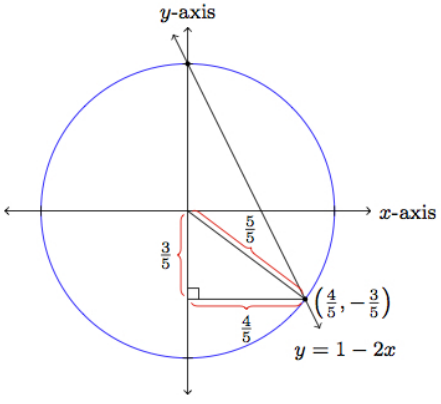
Office Hours

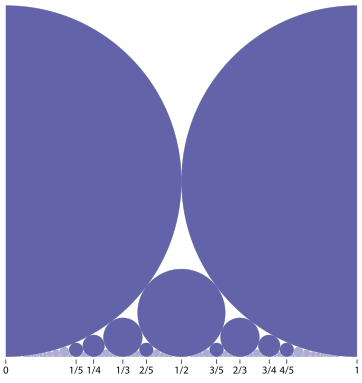
Nathan: Wednesdays 12:30 - 2:30 in DL 418.
Also, feel free to email me to set up an appointment.

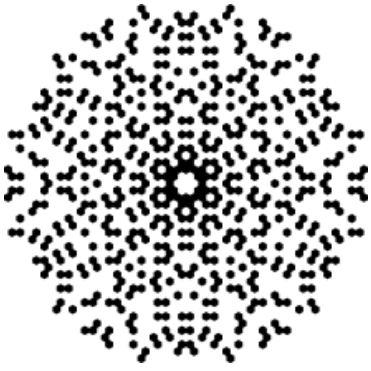
Course Goals

Number theory is one of the oldest and most mysterious parts of mathematics. There are many questions that were posed by the ancient Greeks that we still do not know how to answer today. This course will give an introduction to the area motivated by classifying solutions to different kinds of polynomial equations. For example, how many Pythagorean triples are there? Can we give a recipe for producing them all? Which positive integers can be written as the sum of two square integers? What about four square integers? Which prime numbers can be written as $x^2 + 2y^2$ for nonnegative integers x and y? We will also investigate some challenging questions about the distribution of prime numbers. How many prime numbers are there among the first n positive integers? Are there infinitely many prime numbers that are one more than a multiple of four?

Officially, Math 350 is a prerequisite for this course, but if you are willing to do some outside reading then it is not strictly necessary that you have had a previous course in abstract algebra. As the textbook puts it: "We have assumed some familiarity with the material in a standard undergraduate course in abstract algebra. A large portion of Chapters 1-11 can be read even without such background with the aid of a small amount of supplementary reading." If you have questions about whether you have the appropriate background for the course, I would be happy to meet with you or try to answer your questions by email. You should definitely be comfortable reading and writing proofs.







Grading

- Weekly Homework: 35%
- In-Class Exam 1: February, 24, 20%
- In-Class Exam 2: April, 21, 20%
- Final Assignment: Due April, 30th 25%

There will also be an optional computational project. You can think of the above grading scheme as giving 100 total points. This computational project will be worth up to 10 points. Your final grade will then be taken as a percentage of 110 points. If you choose to do the computational project, it can only help your course grade. That is, if you score on the computational project is lower than your overall course average, it will not be used when computing your grade.

Weekly homework will be a big part of this course. I have always found that I think better about mathematics when I can discuss it with others and that I only really understand a problem when I can explain its solution to somebody else. **You are encouraged to work together on problem sets, but write up your solutions individually. If you use outside sources (other textbooks, websites, etc.) for your homework, you must acknowledge them.**

Course Topics

1. Unique Factorization
2. Congruences
3. Gauss Sums
4. Quadratic Reciprocity
5. The Distribution of Prime Numbers
6. Solving Equations over Finite Fields
7. Sums of Squares
8. Continued Fractions and Pell's Equation
9. Quadratic Forms
10. Public Key Cryptography

Books

The official course textbook is:
A classical introduction to modern number theory, K. Ireland and M. Rosen. ISBN: 978-1-4419-3094-1. This book is available for free online for Yale students via Springerlink.

There are other books that will be useful for certain parts of the course For example, I also like *Elementary methods in number theory* by Nathanson, and *An introduction to the theory of numbers* by Niven, Zuckerman, and Montgomery. Neither of these books is required.