# Math 719: Asymptotic Problems in Number Theory

**Spring 2015 Course Information and Syllabus**

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**Lectures: Tu,Th 1:00 - 2:15 in LOM 206.**
**Office Hours**: We 12:30 - 2:30.

## Course Outline

In this course we will focus on properties of families of arithmetic objects. For example, instead of discussing the algorithms that go into computing the ideal class groups of a particular number field, we will study the average behavior of class group as we range through infinite collections of fields. This subject, sometimes called arithmetic statistics, has become very popular in recent years. If we have some infinite family of arithmetic objects, how can we develop a reasonable definition for what it means for this family to be 'good' or 'extremal'? Often the study of zeta functions attached to these objects will be quite helpful. We will consider these questions in the context of number fields, curves over finite fields, and linear codes.

1. Class Groups of Number Fields
    - The Dedekind class number formula, 2-parts of class groups and genus theory, 3-parts and the Davenport-Heilbronn Theorem, counting number fields, the Brauer-Siegel Theorem, Cohen-Lenstra heuristics.
2. Curves over Finite Fields
    - The zeta function of an algebraic curve, the Weil conjectures for curves, improvements to the Weil bound, low genus examples, elliptic curves and Birch's theorem, Hermitian curves, modular curves.
3. Coding theory
    - Basic coding theory, Gilbert-Varshamov bound, Reed-Solomon and Reed-Muller codes, Algebraic geometry codes, Tsfasman-Vladut-Zink Theorem, the JPL bound.
4. Possible other topics include: Sphere packings and lattices, and regular graphs.

This course will require some background in both number theory and in algebraic geometry. If you are unsure of whether you are ready for such a course I would be happy to meet with you and to suggest some additional reading.

Course participants will select topics related to the course material and give short presentations.