Math 206A: Algebra
Homework 5

Due Friday, November 13th at 12PM.
Please email `nckaplan@math.uci.edu` with questions.

The last exercise comes from a UCI Comprehensive Exam. All other exercises are from Dummit and Foote.

1. Exercise 4 Section 5.1.
   In this exercise you work out some key facts about Sylow p-subgroups in direct products.

2. Exercise 15 of Section 5.1.
   This is the exercise I mentioned in lecture about taking the direct product of groups with an arbitrary (possibly infinite) index set I.

3. Exercise 17 of Section 5.1.
   This is the exercise I mentioned in lecture about the restricted direct product, a special subgroup of the direct product described in Exercise 15.

4. Exercise 18 of Section 5.1.
   This exercise asks you to give examples of several types of behavior that sound like maybe they should not be possible at first. It is good to be aware of these kinds of examples since they often come up as True/False questions on Comprehensive and Qualifying Exams.

5. Exercises 2 and 3 of Section 5.2.
   These two exercises ask you to go back and forth between invariant factors and elementary divisors for abelian groups of given order in some examples.

6. Exercises 5 and 6 of Section 5.2.
   These two exercises introduce some of the basics of the exponent of a group.

7. Exercise 7 of Section 5.2.
   This exercise introduces the p^{th}-power map on a finite abelian group. In Lecture 20 we will give a version of the proof of Part (2) of the Primary Decomposition Theorem that is given at the end of Section 6.1. On page 198, Dummit and Foote indicate that one can prove the uniqueness part of the Primary Decomposition by induction using this p^{th}-power map.

8. Exercise 8 of Section 5.2.
   This exercise goes into more detail about the p^{th}-power map. Part (b) is a nice fact that is the beginning of a much larger story about counting subgroups of given order/index in a finite abelian group. It is related to Exercise 14 of Section 5.2 (which you do not have to do, but might want to look at).

9. Spring 2015 Comprehensive Exam #3:
   Let A, B, and C be finitely generated abelian groups such that $A \times C \cong B \times C$. Prove that $A \cong B$.