Math 206A: Algebra
Homework 7

Due Friday, December 11th at 11:59PM.
Please email `nckaplan@math.uci.edu` with questions.

All exercises are from Dummit and Foote.

1. Exercise 6 of Section 7.1.
   In this exercise you determine whether several subsets of the ring of all functions from $[0,1]$ to $\mathbb{R}$ are subrings or not.

2. Exercises 13 and 14 of Section 7.1.
   The first exercise introduces the concept of a *nilpotent* element of a ring and asks you to work through some basic examples. The next exercise builds on this one.

3. Exercise 17 of Section 7.1.
   This exercise introduces the idea of the direct product of two rings.
   (You may also want to look at Exercises 19 and 20 which introduce the idea of taking the direct product $\prod_{i \in I} R_i$ of an arbitrary collection of rings, and the idea of taking the *direct sum* of a collection of rings. You do not need to hand these in.)

4. Exercises 3 and 4 of Section 7.2.
   The first exercise introduces the concept of the ring of *formal power series*, which plays an important role in several parts of math (including combinatorics). The next exercise has you prove a basic property of this ring.

5. Exercise 5 of Section 7.2.
   This exercise introduces *Laurent series*, another nice example to go along with polynomial rings and formal power series rings. The second part of this problem uses results of Exercise 26 of Section 7.1. You can use these results without proving them.

6. Exercise 6 of Section 7.3.
   In this exercise you determine whether several functions involving 2×2 matrices are ring homomorphisms or not.

7. Exercises 8 and 9 of Section 7.3.
   In these exercises you determine whether or not several subsets of various rings are ideals or not.

8. Exercise 17 of Section 7.3.
   This exercise emphasizes some of the differences that result from the definition of a ring homomorphism given in Dummit and Foote and the definition given in Conrad's "Standard Definitions for Rings" notes.
   (We already proved the first part of (b) in Lecture 27.)

9. Exercise 18 of Section 7.3.
   In this exercise you prove that the intersection of ideals is again an ideal.