Math 206A: Algebra
Additional Midterm 2 Practice Problems

Here are some additional practice problems to help you prepare for Midterm 2.

1. Lemma 6.2 of Conrad's 'Semidirect Product' notes:
   A semidirect product $H \rtimes_\varphi K$ is unchanged up to isomorphism if the action $\varphi\colon K \to \mathrm{Aut}(H)$ is composed with an automorphism of K: for automorphisms $f\colon K \to K$, $H \rtimes_{\varphi \circ f} K \cong H \rtimes_\varphi K$.

2. Spring 2019 Comprehensive Exam #1:
   Let p, q denote distinct primes. Assume G is a finite group, and assume that G has a unique Sylow p-subgroup and also a unique Sylow q-subgroup. Assume $g_1 \in G$ has order p and $g_2 \in G$ has order q. Prove that $g_1 g_2 = g_2 g_1$.

3. Fall 2006 Advisory Exam #2:
   Let G be a finite group and let $H, K \leq G$ such that $HK \leq G$.
   (a) If $h \in H$ and $k \in K$ show that $|hk|$ divides $|H| \cdot |K|$.
   (b) Let $N \leq G$ be such that $|N|$ is relatively prime to $|H| \cdot |K|$. Prove that $HN = KN$ implies $H = K$.

4. Spring 2009 Comprehensive Exam #1:
   Show that every group of order 12 is solvable.

5. Spring 2009 Comprehensive Exam #10:
   Show that there is no simple group of order $858 = 2 \cdot 3 \cdot 11 \cdot 13$.

6. Spring 2014 Comprehensive Exam #3:
   Classify all groups of order $2014 = 2 \cdot 19 \cdot 53$.
   Hint: Show that there is a normal subgroup isomorphic to $\mathbb{Z}/19\mathbb{Z} \times \mathbb{Z}/53\mathbb{Z}$ and then observe that conjugation by an element of order two induces an order two automorphism of this subgroup.

7. Spring 2013 Comprehensive Exam #3:
   Show that a group of order 340 has a cyclic subgroup of order 85.

8. Spring 2013 Comprehensive Exam #1:
   Show that no group of order 825 is simple.

9. Spring 2019 Qualifying Exam #1:
   Does the symmetric group S_5 contain a subgroup isomorphic to:
   (a) The dihedral group D_8 with 8 elements?
   (b) The quaternion group Q_8 with 8 elements?

10. Fall 2016 Qualifying Exam #1:
    Is every group of order 39 cyclic?
    Either prove this or construct a non-cyclic group of order 39.

11. Fall 2016 Qualifying Exam #2:
    Let H be a subgroup of S_p of order p. What is $|N_{S_p}(H)|$, the order of the normalizer of H?

12. Fall 2010 Advisory Exam #3:
    Show that for any integer $n \geq 1$ the quotient group $\mathbb{Q}/\mathbb{Z}$ has a unique subgroup of order n.