Math 206B: Algebra
Homework 5

Due Friday, February 19th at 12:00PM.
Please email `nckaplan@math.uci.edu` with questions.

1. Exercise 5 of Section 10.1
   In this exercise you see how an ideal I of an R-module M gives a submodule IM of M.

2. Exercise 8 of Section 10.1 and Exercise 8 of Section 10.2
   The first exercise introduces the important example of the torsion submodule of an R-module M. The next exercise shows how this torsion submodule behaves under an R-module homomorphism.

3. Exercises 9 and 10 of Section 10.1
   In this pair of exercises you see how a submodule N of an R-module M leads to a certain ideal of R, and how an ideal I of R leads to a certain submodule.

4. Exercises 11 and 12 of Section 10.1
   These two exercises build on Exercises 9 and 10. Exercise 11 gives an example involving finite abelian groups. In Exercise 12 you provide some examples that will be helpful to keep in mind.

5. Exercise 4 of Section 10.2
   In this exercise you determine when a certain map gives a $\mathbb{Z}$-module homomorphism, and as a consequence you compute $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}/n\mathbb{Z}, A)$ where A is an $\mathbb{Z}$-module.

6. Exercise 6 of Section 10.2
   Proposition 2 in Section 10.2 says that $\mathrm{Hom}_{\mathbb{Z}}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$ is an abelian group. In this exercise, you determine which abelian group it is isomorphic to.

7. Exercise 10 of Section 10.2
   Proposition 2 in Section 10.2 says that when R is a commutative ring, $\mathrm{Hom}_R(R, R)$ is also a ring. In this exercise you determine which ring it is isomorphic to.