Math 206B: Algebra
Homework 6

Due Friday, March 12th at 11:59PM.
Please email `nckaplan@math.uci.edu` with questions.

The focus of this homework is on the fundamentals of vector spaces, Sections 11.1 and 11.2 of Dummit and Foote. You have probably proven some of these facts before in a first linear algebra course, but probably only in the case where $F = \mathbb{R}$.

1. Exercise 1 of Section 11.1: In this exercise you show that the set of vectors satisfying a fixed linear equation forms a subspace. In 'algebraic geometry language' this set of vectors is a 'hyperplane' in $\mathbb{R}^n$.

2. Exercise 6 of Section 11.1: In this exercise you consider a linear transformation $\varphi\colon V \to V$ and its iterates $\varphi^m = \varphi \circ \cdots \circ \varphi$. More specifically, you look at what happens to the image and kernel of φ as you iterate.

3. Exercise 8 of Section 11.1: This exercise defines *eigenvector* and *eigenvalue* and asks you to prove that the collection of eigenvectors with a fixed eigenvalue together with the 0 vector forms a subspace.

4. Exercise 9 of Section 11.1: This exercise asks you to show that collections of eigenvectors with distinct eigenvalues are linearly independent.

5. Exercises 10 and 11 of Section 11.1: I mentioned these two exercise in lecture. We have proven that a vector space of dimension n over F contains a basis, and that moreover, any collection A of linearly independent vectors in V can be extended to a basis. In these exercises you prove the analogous results for vector spaces of arbitrary dimension over F.

6. Exercise 5 of Section 11.2: This is another exercise that I mentioned in lecture. Here you give a precise relation between the concept of *nonsingular* for a linear transformation and for a matrix.

7. Exercise 6 of Section 11.2: I mentioned this exercise in lecture when I defined the column rank of a matrix. Here you prove a relation between the rank of a linear transformation φ and the column rank of a matrix representing φ.

8. Exercise 9 of Section 11.2: In this exercise you show that if $\varphi\colon V \to V$ is a linear transformation and W is a subspace of V that is stable under φ, then the restriction of φ to W is a linear transformation on W, and φ defines a linear transformation on the quotient V/W.