Math 206C: Algebra
Homework 1

Due Wednesday, April 14th at 11:59PM.
Please email `nckaplan@math.uci.edu` with questions.

For a ring R, let $\mathrm{Mat}_n(R)$ denote the ring of $n \times n$ matrices with entries in R.

1. Algebra Qualifying Exam Fall 2017 #4
   Determine up to isomorphism all $\mathbb{F}_2[x]$-modules of order 4.

2. Algebra Comprehensive Exam Spring 2016 #10
   Prove that for a matrix $A \in \mathrm{Mat}_n(\mathbb{R})$, the minimal and characteristic polynomial of A coincide if and only if there is a basis of $\mathbb{R}^n$ of the form $\{v, Av, A^2v, \ldots, A^{n-1}v\}$.

3. Algebra Comprehensive Exam Spring 2018 #4
   The group $\mathrm{GL}_2(\mathbb{C})$ acts on $\mathrm{Mat}_2(\mathbb{C})$ by conjugation. Classify the orbits of this action. (For example, you could give a list of representatives for the orbits, with one representative for each orbit.)

4. Algebra Comprehensive Exam Fall 2012 #7
   Classify, up to conjugation, all 4×4 real matrices with minimal polynomial $(x^2 + 4)(x - 1)$.

5. Exercise 10 of Section 12.2
   This exercise is very similar to Example (4) from pages 486-487 that we discussed in Lecture 4.

6. Exercise 11 of Section 12.2
   This exercise builds on Example (4) from pages 486-487 but over $\mathbb{C}$ instead of over $\mathbb{Q}$.
   We mentioned this exercise in Lecture 4.

7. Exercise 15 of Section 12.2
   This exercise is similar to Example (5) from page 487-8. We mentioned in Lecture 4 that this example tells you about matrices in $\mathrm{GL}_3(\mathbb{Q})$ of order dividing 6. By also considering matrices of order 3 and 2, you can classify matrices of order **exactly** 6. In this exercise you do something similar for matrices in $\mathrm{Mat}_2(\mathbb{Q})$ and $\mathrm{Mat}_2(\mathbb{C})$ of order 4.

8. Exercise 17 of Section 12.2
   In this exercise you apply the ideas developed in previous exercise for matrices with entries in $\mathbb{Q}, \mathbb{R}$, and $\mathbb{C}$, to matrices with entries in a finite field.

9. Exercise 18 of Section 12.2
   In this exercise you show that if a linear transformation $T\colon V \to V$ satisfies a certain constraint, then we can deduce something about the dimension of V.