Math 206C: Algebra
Homework 3

Due Monday, May 3rd at 11:59PM.
Please email `nckaplan@math.uci.edu` with questions.

1. Algebra Comprehensive Exam Spring 2020 #9
   Let E/F be a field extension of degree 12.
   Prove that there exist $\alpha_1, \alpha_2, \alpha_3 \in E$ such that $E = F(\alpha_1, \alpha_2, \alpha_3)$.

2. Exercise 5 of Section 13.1
   This exercise highlights the difference between the set of solutions of polynomials in $F[x]$ and the set of solutions of **monic** polynomials in $F[x]$.

3. Exercise 6 of Section 13.2.
   In lecture we showed how to understand $F(\alpha_1, \ldots, \alpha_n)$ via a sequence of simple extensions. In this exercise you prove that extension is the composite of the fields $F(\alpha_1), \ldots, F(\alpha_n)$.

4. Exercise 7 of Section 13.2
   This exercise is very similar to an example from lecture.

5. Exercise 8 of Section 13.2
   This exercise builds on our discussion of quadratic extensions from lecture to define another interesting class of examples that are useful to keep in mind, *biquadratic extensions*.

6. Exercise 13 of Section 13.2
   This exercise gives an example application of the material we have developed about degrees of field extensions generated by finite sets of algebraic elements.

7. Exercise 14 of Section 13.2
   This exercise gives another nice application of results about multiplicativity of degrees of field extensions.
   Note: This also came up as Spring 2014 Algebra Comprehensive Exam #9.

8. Algebra Comprehensive Exam Spring 2019 #8
   Let $f(x) \in \mathbb{Q}[x]$ be irreducible of odd degree.
   If α is a root of $f(x)$, prove that $\mathbb{Q}(\alpha) = \mathbb{Q}(\alpha^{2^k})$ for all positive integers k.
   Note: This exercise is quite similar to the previous one.

9. Exercise 16 of Section 13.2
   We have talked a lot about subfields of a given field. In this exercise you show that if R is a ring sandwiched between two fields, then it is automatically a field.