Math 206C: Algebra
Midterm 1 (Canonical Forms): Things to Know

The goal of this document is to give a list of definitions and theorems related to our unit on Canonical Forms (Sections 12.2 and 12.3 of Dummit and Foote) that will be helpful to know for Midterm 1 on Friday, April 23. This document does not contain an Examples section, but I would like to emphasize that we went through several problems with particular linear transformations/matrices in lectures, and it would be good to review these.

Canonical Forms

Definitions

1. Minimal polynomial of a linear transformation and of an $n \times n$ matrix.
2. Characteristic polynomial of a linear transformation and of an $n \times n$ matrix.
3. Eigenvalues and Eigenvectors of a linear transformation and of an $n \times n$ matrix.
4. Eigenspace corresponding to the eigenvalue λ of a linear transformation. Eigenspace corresponding to the eigenvalue λ of an $n \times n$ matrix.
5. Determinant of a linear transformation.
6. Companion matrix of a monic polynomial $a(x)$.
7. What is means for a matrix to be in rational canonical form. The rational canonical form of a linear transformation.
8. Invariant factors of a linear transformation and of an $n \times n$ matrix.
9. Elementary divisors of a linear transformation and of an $n \times n$ matrix.
10. Elementary row and column operations and the Smith Normal Form of a matrix.
    (Theorem 21 in Section 12.2)
11. $k \times k$ elementary Jordan matrix with eigenvalue λ/Jordan block of size k with eigenvalue λ.
12. Jordan canonical form of a linear transformation and of an $n \times n$ matrix.

Theorems

1. Theorem 5 of Section 12.1 applied to a finite dimensional $F[x]$-module V.
   (Invariant Factor Form)
2. Theorem 6 of Section 12.1 applied to a finite dimensional $F[x]$-module V.
   (Elementary Divisor Form)

3. Equivalent conditions for λ to be an eigenvalue of a linear transformation T. (Proposition 12 in Section 12.2)
4. Similar matrices have the same characteristic polynomial and the same minimal polynomial.
5. The existence and uniqueness of the rational canonical form for a linear transformation. (Theorem 14 in Section 12.2)
6. The existence and uniqueness of the rational canonical form for an $n \times n$ matrix A. (Theorem 16 in Section 12.2)
7. The relationship between similarity and rational canonical form. (Theorem 17 in Section 12.2)
8. Let F be a field that is a subfield of a field K. The relationship between the rational canonical form of an $n \times n$ matrix A with entries in F and the rational canonical form of A considered as a matrix with entries in K. (Corollary 18 in Section 12.2)
9. The characteristic polynomial is the product of all of the invariant factors. The Cayley-Hamilton theorem. The characteristic polynomial divides some power of the minimal polynomial. (Proposition 20 in Section 12.2.)
10. Invariant factors of 2×2 matrices. Two 3×3 matrices with the same minimal and characteristic polynomial have the same invariant factors. Example: 4×4 matrices with the same minimal and characteristic polynomials that **do not** have the same invariant factors. (Lecture 3)
11. Interpretations of the constant coefficient and the x^{n-1} coefficient of the characteristic polynomial of an $n \times n$ matrix. (Lecture 3)
12. The Invariant Decomposition Algorithm and the Elementary Divisor Decomposition Algorithm.
13. Determine all similarity classes of matrices A with entries in F with a given characteristic polynomial.
14. Determine all similarity classes of $n \times n$ matrices A with entries in F satisfying a given equation, (Example: $A^6 = I$).
15. The existence and uniqueness (up to permutation of the blocks) of the Jordan canonical form for a linear transformation.
16. The existence and uniqueness (up to permutation of the blocks) of the Jordan canonical form for an $n \times n$ matrix.
17. A is similar to a diagonal matrix over F if and only if the minimal polynomial of A has no repeated roots.
18. The algorithm to change from one canonical form to another.