Math 206C: Algebra
Midterm 1 Practice Problems: Fields

The goal of this document is to provide you with some practice problems for Midterm 1 about Fields. These problems are a mix of Exercises from Dummit and Foote and problems from past Algebra Comprehensive and Advisory Exams.

1. Exercises 1 and 2 Section 13.1
   These two exercises ask you to compute the inverse of some element in a field of thee form $F[x]/(p(x))$.

2. Exercise 3 Section 13.1
   In this exercise you compute the powers of an element in $\mathbb{F}_2[x]/(x^2+x+1)$.

3. Exercise 4 Section 13.1
   In this exercise you prove that a certain map is an isomorphism of $\mathbb{Q}(\sqrt{2})$ with itself.

4. Exercise 1 Section 13.2
   In this exercise you prove a basic fact about finite fields (that we have stated before).
   Note: This also came up as Algebra Advisory Exam Fall 2008 #F7.

5. Exercise 2 Section 13.2
   In this exercise you are asked to construct some finite fields. We wrote down the multiplication table for a field with 4 elements in lecture, and in this exercise you do the same for a field with 9 elements.

6. Exercises 3 and 7 Section 13.2
   In these exercises you write down the minimal polynomials for some elements.

7. Exercise 4 Section 13.2
   In this exercise you compute the degree of some field extensions.

8. Exercise 5 Section 13.2
   This exercise is similar to something we discussed in lecture, where we considered the minimal polynomial of $\sqrt[6]{2}$ over $\mathbb{Q}(\sqrt{2})$.

9. Algebra Comprehensive Exam Spring 2010 #F9
   Let $E \subset F$ be a field extension of degree 5 and let $a \in F$ be an element that is a root of some cubic polynomial $g(x) \in E[x]$. Show that $a \in E$.

10. Algebra Comprehensive Exam Spring 2008 #F6

    (a) If E/F is a field extension of finite degree, prove that E is algebraic over F.

    (b) Let E/F be a field extension. If $a, b \in E$ are algebraic over F, show that $a+b$ is also algebraic over F.

    Note: The first part of this question is Corollary 13 in Section 13.2, which we proved in lecture. The second part is Corollary 18 in Section 13.2, which we did not state in lecture but follows directly from Theorem 17, which we proved in the last video of Lecture 10. Both of these parts have come up several times on Comprehensive Exams (see Spring 2007 #F1, Spring 2005 #F1, Spring 2006 #F8, Spring 2016 #8).