# Math 230a: Algebra

## In-Class Exam 1

Friday, October 23 2015.

**NAME**:

- You have 50 minutes for this exam. Pace yourself, and do not spend too much time on any one problem.
- Show your work and justify all of your answers. The more you explain your thought process, the easier it will be to give partial credit for incomplete solutions.
- This is a closed-book exam. No notes or outside resources can be used. Do not use a calculator.
- If you need more room, use extra pages, and indicate clearly that you have done so.
- You may use results that we proved in lecture or on the homework without proving them here provided you clearly state the result your are using.

| **Problems** | |
|---|---|
| **1** | |
| **2** | |
| **3** | |
| **4** | |
| **5** | |
| **6** | |
| **Total** | |

1. (a) Let $G$ be a group and $X$ a set. Explain what it means to have a left group action of $G$ on $X$.

   (b) True/False (Circle One): Let the set $X$ be equal to $G$. The map that takes a pair $(g, x)$ to $g^{-1}x$ defines a left group action.

2. State the Second (Diamond) Isomorphism Theorem.

3. (a) Define what it means for a group $G$ to be solvable.

(b) For which $n \geq 3$ is the dihedral group of order $2n$, $D_{2n}$, solvable? Justify your answer.

4. Suppose that $H$ and $K$ are subgroups of a group $G$. Prove that if $H \cup K$ is a subgroup, then either $H \subseteq K$ or $K \subseteq H$.

5. Determine all finite groups that have at most three conjugacy classes.

6. (a) Prove that conjugate elements of a group $G$ have the same order.

(b) Is the converse true? Justify your answer.