MATH 13 HOMEWORK 4
DUE: Wednesday, May 2

READING ASSIGNMENT: Read Sections 4.1,4.2, 4.3, 4.4 of the course notes.
PROBLEMS FROM COURSE NOTES: Do problems 4.1.1, 4.1.2, 4.1.5, 4.1.6, 4.2.2, 4.2.5, 4.3.1, 4.3.5, 4.4.1, 4.4.3, 4.4.6, 4.4.9
ADDITIONAL PROBLEMS ON PROOFS BY MINIMUM COUNTER EXAMPLES:

Problem 1. Read the following wikipedia page on the proof of the unique prime factorization theorem we stated as a fact in class (this is typically known as the Fundamental Theorem of Arithmetic):

`https://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic`

Problem 2. The goal of this exercise is to prove Theorem 3.13 in the notes. Let a, b, c be integers and $d = gcd(a, b)$.

(a) Recall we have shown in class that the equation $ax + by = d$ has integer solutions (using the Euclidean algorithm). Use this fact to show that $ax + by = c$ has integer solutions iff $d|c$.

(b) Show that (x_0, y_0) is a solution to $ax + by = 0$ iff (x_0, y_0) is a solution to $\frac{a}{d}x + \frac{b}{d}y = 0$.

(c) Show that if (x_0, y_0) and (x_1, y_1) are solutions to $ax+by = c$, then (x_0-x_1, y_0-y_1) is a solution to $ax + by = 0$.

(d) Use (b) to show that all integer solutions to $ax + by = 0$ have the form:

$$x = \frac{b}{d}t \qquad y = -\frac{a}{d}t \qquad \text{where } t \in \mathbb{Z}.$$

Hint: First show for any $t \in \mathbb{Z}$, $(\frac{b}{d}t, -\frac{a}{d}t)$ is an integer solution to $ax+by = 0$. Then show any integer solution to $ax + by = 0$ has the form $(\frac{b}{d}t, -\frac{a}{d}t)$ for some $t \in \mathbb{Z}$.

(e) Now assume $d|c$. Use parts (c), (d) to show that if (x_0, y_0) is a solution to $ax + by = c$ (this exists by part (a)), then all integer solutions to $ax + by = c$ have the form

$$x = x_0 + \frac{b}{d}t \qquad y = y_0 - \frac{a}{d}t \qquad \text{where } t \in \mathbb{Z}.$$