MATH 13 HOMEWORK 1 ANSWER KEY

Problem 2.1.10:

(a) Contrapositive: "If someone was playing pool, then Colin was late". This is a true statement (as it is logically equivalent to the original statement).

(b) Converse: "If no-one was playing pool, then Colin was early". We do not know if this is a true statement (generally, knowing the truth of the original statement does not tell us anything about the truth/falsity of the converse).

(c) For (i): if we know "someone was playing pool" is TRUE, then we know Colin was late (because the contrapositive in (a) is true). For (ii): if we know "Colin was late" is TRUE, we cannot conclude anything else

Problem 2.2.7(a):

($\Rightarrow$): Assume $5x + 3$ is even. We show $7x - 2$ is odd.

Note that since 3 is odd and $5x + 3$ is even, $5x$ must be odd. Hence x must be odd (Why? If x was even, say $x = 2k$ for some k, then $5x = 2(5k)$ is even. Contradiction.). Let l be such that $x = 2l + 1$. Then $7x - 2 = 7(2l + 1) - 2 = 2(7l + 2) + 1$ is clearly odd.

($\Leftarrow$): Now assume $7x - 2$ is odd. We show $5x + 3$ is even.

We have that $7x$ must be odd and hence x is odd (by a similar argument as above). Let $x = 2l + 1$ for some l. Then $5x + 3 = 5(2l + 1) + 3 = 2(5l + 4)$ is even.

Problem 3:

The original statement can be written as "$x \geq 10 \wedge y \geq 10$". Now use DeMorgan's law to negate this statement, we get the negation is: "$x < 10 \vee y < 10$". Translate this back to English: "x is less than 10 or y is less than 10".

Problem 4b: Suppose n is not divisible by 3. There are two cases:

Case 1: $n = 3k + 1$ for some integer k. Then $n^2 - 1 = (3k + 1)^2 - 1 = 9k^2 + 6k + 1 - 1 = 9k^2 + 6k = 3(3k^2 + 2k)$ is clearly divisible by 3.

Case 2: $n = 3k + 2$ for some integer k. Then $n^2 - 1 = (3k + 2)^2 - 1 = 9k^2 + 6k + 4 - 1 = 9k^2 + 6k + 3 = 3(3k^2 + 2k + 1)$ again is divisible by 3.

Problem 6: We simply examine the truth table of $(P \wedge Q) \Rightarrow (P \vee Q)$. You can write out the full truth table for this; it is basically the same as what I'm doing here.

Observe that if P is T then $P \vee Q$ is T. Hence $(P \wedge Q) \Rightarrow (P \vee Q)$ is T (regardless of the truth value of Q).

If P is F, then $P \wedge Q$ is F, hence $(P \wedge Q) \Rightarrow (P \vee Q)$ is T(regardless of the truth value of Q).

Hence $(P \wedge Q) \Rightarrow (P \vee Q)$ is T in all cases. Therefore, it is a tautology.