MATH 150 HOMEWORK 2

DUE: Friday, Oct 20
Student Name/ID # (Include all students in the group):

IMPORTANT INSTRUCTION: It is crucial that you clearly explain how you arrive at your conclusions. The point of the homework is to practice understanding of the material and the ability to express your understanding.

1. (**5 pts**) In the lecture we gave a recipe for the following task: Given an n-place Boolean function $F : \{T, F\}^n \to \{T, F\}$, find a formula ϕ in the disjunctive normal form built from sentential symbols $A_1, \ldots, A_n$ such that $F = B^n_\phi$. In this exercise we do the dual task. First, **a conjunctive normal form** is the form defined dually to disjunctive normal form. Here "dually" means that we write conjunctions in place of disjunctions and vice versa.

(a) **(1pt)** Give a rigorous formulation of the notion of conjunctive normal form. That is, express when a formula ϕ is in a conjunctive normal form. For this, dualize the definition of disjunctive normal form from the lecture.

(b) **(2pt)** Describe a recipe which performs the following task. Given an n-place Boolean function $F : \{T, F\}^n \to \{F, T\}$, the recipe produces a formula ϕ in the conjunctive normal form built from sentential letters $A_1, \ldots, A_n$ such that $F = B^\phi_n$. To find such a recipe, dualize the recipe for obtaining disjunctive normal form from the lecture. Caution: You need to dualize not only the syntax, but also semantics. So for instance the roles of values 0 and 1 need to be swapped at some steps in the recipe. Figure out where.

(c) **(2pt)** Apply the recipe you described in (b) to find a formula ϕ in conjunctive normal form such that $F = B^\phi_3$ where $F : \{T, F\}^3 \to \{T, F\}$ is the Boolean function such that $F(F, T, T) = F(F, F, T) = F$ and $F(a, b, c) = T$ otherwise.

2. (**5 pts**) Determine if the following sets of connectives are complete. In the positive case, you are allowed to show completeness by reducing to one of the complete sets $\{\neg, \wedge\}$, $\{\neg, \vee\}$. In the negative case find some peculiarity of the given set of connectives which prevents it from being complete, and explain how this peculiarity yields incompleteness. If appropriate, prove by induction on complexity of formulas that the peculiarity is preserved under forming formulas using the connectives from the given set.

(a) **(2pt)** $\{\neg, \rightarrow\}$.

(b) **(2pt)** $\{\neg \leftrightarrow\}$.

(c) **(1pt)** $\{\downarrow\}$, where $\downarrow$ is a binary connective defined as: $A \downarrow B = \neg(A \vee B)$.

3. (**5pt**) In the following, given an n-Boolean function G, find a wff α such that $B^n_\alpha = G$. You are allowed to use connectives $\{\neg, \wedge, \vee\}$ to build α and you must ensure that the delay for α is **no more than 7**.

(a) **(3pt)** $G : \{T, F\}^4 \rightarrow \{T, F\}$ is such that $G(F, T, F, F) = G(T, T, F, F) = G(F, F, F, T) = G(F, F, T, T) = G(T, F, F, T) = G(T, F, T, T) = F$ and $G(a, b, c, d) = T$ otherwise.

(b) **(2pt)** $G : \{T, F\}^3 \rightarrow \{T, F\}$ is such that $G(F, T, F) = G(T, T, F) = G(F, F, T) = G(T, F, T) = T$ and $G(a, b, c) = F$ otherwise.