MATH 3A FINAL REVIEW

1. DEFINITIONS, CONCEPTS, AND THEOREMS

Row echelon form and row reduced echelon form, linear independence/dependence of vectors, linear transformations and related notions (domain, range, codomain, one-to-one, onto), matrix operations (addition, scalar/matrix multiplication, transpose, and inverse) and elementary matrices, subspace of $\mathbb{R}^n$, dimension/basis of a subspace, rank/nullity of a matrix, determinant of a matrix + Lagrange theorem, eigenvalues/eigenvectors and characteristic equation of a matrix, similarity of 2 matrices, diagonalizability.

Theorem 4 in Section 1.4, Theorem 10 in Section 1.9, the Invertible Matrix Theorem (in various sections), Rank + Nullity theorem (Section 2.9), Theorems 5,6,7 of Section 5.3, Theorem 9 of Section 5.5.

2. TOPICS AND PROBLEMS

2.1. COMPUTATIONAL

1. Solving systems of linear equations and matrix equations $A\vec{x} = \vec{b}$ (row reduction to REF, determining whether the system has solutions/has no solutions/has unique solutions based on the pivots of REF).
2. Converting matrix equations of the form $A\vec{x} = \vec{b}$ into a system of linear equations and vice versa.
3. Being able to express solutions to $A\vec{x} = \vec{b}$ in parametric form (Theorem 6 of Section 1.5).
4. Determining if a set of vectors $\{v_1, \ldots, v_n\}$ is linearly independent and if a vector v is in the span of other vectors (e.g. Problems 1-14 in Section 1.7).
5. Computing the standard matrix for a given linear transformation T (see Theorem 10 of Section 1.9 and relevant homework problems).
6. Computing basic matrix operations (addition, transpose, multiplication), and inverse A^{-1} using row reduction (see Theorem 7 of Section 2.2).
7. Find basis for Column space/null space of a matrix (see examples 6,7 of Section 2.8 and relevant problems in the section).
8. Computing determinant of a matrix (using Lagrange theorem and using row reductions) and applications to computing area/volume and how linear transformations change area/volume (consult relevant homework problems).

9. Computing characteristic equation of a matrix and computing the (real or complex) eigenvalues/eigenspaces/basis for eigenspace of a matrix and being able to tell whether a matrix is diagonalizable (and if it is, diagonalize it) (consult relevant homework problems). **Make sure you practice factoring, finding roots of characteristic equations, especially for 2x2 and 3x3 matrices**.
10. Study Section 5.5 (besides the homework problems, work through problems 13-20).
11. Study the first half of Section 5.7 (up to and not including Decoupling a Dynamical System). Work through problems 1-6 in Section 5.7.

2.2. CONCEPTUAL

1. Showing some function T is a linear transformation (example 4 in Section 1.8 and relevant problems in the same section) and showing if a function/linear transformation T is one-to-one, onto (see Theorem 12 Section 1.9).
2. Applications of invertible matrix theorem (e.g. 13-33 of Section 2.3).
3. Showing some subset of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.
4. Applications of Rank + Nullity Theorem (Problems 19-26 of Section 2.9).
5. Properties of determinants $(det(A.B) = det(A)det(B), det(A^T) = det(A), det(A + B) \neq det(A) + det(B)$ etc.) (e.g. Problems 31-36 in Section 3.2).
6. Conceptual problems regarding eigenvalues/eigenspaces of a matrix and diagonalizability (e.g. 21-30 of Section 5.1, 21-32 of Section 5.3).