# Math 161 Sample Final

(1) (10 pts) Show that two hyperbolic lines cannot have more than one common perpendicular.

(2) (10 pts) Draw a cevian line for a triangle $ABC$ (in hyperbolic geometry). Prove that the angle defect ($\pi$ radians minus the sum of the angles in the triangle) is equal to the sum of the defects of the two sub-triangles created by the cevian line.

(3) (10 pts) Prove that two Saccheri quadrilaterals with equal bases and equal summit angles must be congruent.

(4) (25 pts) In the Poincare model, find (equation of) the hyperbolic line $(l)$ through the points $A = (0, \frac{1}{2})$ and $B = (\frac{1}{4}, 0)$.

  (a) Find the Omega points of $(l)$ and the hyperbolic distance between the two points $A, B$.

  (b) Let $P = (\frac{1}{2}, 0)$. Find the limiting parallels through $P$ to the line $(l)$.

(5) (15 pts) Suppose $f$ is a rotation about the origin by angle $\theta$ and $g$ is a reflection about the line $(l)$ through the origin making able $\gamma$ with the positive $x$-axis. Specify what type of maps $g \circ f$ and $f \circ g$ are (by computing their matrices). Is $f \circ g = g \circ f$?

(6) (20 pts) Identify the product $T$ of the reflection through the line $x - y = 2$ followed by rotation by $\pi/2$ around the point $(0, 1)$. Compute $T(1, 1)$.

(7) (15 pts) Let $c$ be the circumscribed circle of $\triangle ABC$ and let $P$ be the point on $c$ where the bisector of $\angle ABC$ meets $c$. Let $O$ be the center of $c$. Prove that the radius $OP$ meets $AC$ at right angles.

(8) (10 pts)

  (a) Show that $i^{-2i}$ is a complex number. Compute the angle this number makes with the positive real axis of the complex plane.

  (b) Show that for any real numbers $a, b, c$,
  $$ab + bc + ca \leq a^2 + b^2 + c^2.$$
  And equality is achieved exactly when $a = b = c$.