

(3rd)

$$\rightarrow \cos(\theta - \varphi) = 0$$

$$\Rightarrow \varphi = \frac{\pi}{2} + \theta \rightsquigarrow b = \sin \theta, d = -\cos \theta$$

$$\varphi = \frac{3\pi}{2} + \theta \rightsquigarrow b = -\sin \theta, d = \cos \theta$$

$$\text{So } A = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \text{ or } \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

reflection

rotation

Generally:

- Then:
- $R_\alpha R_\beta = R_{\alpha+\beta}$
 - $R_\alpha M_\beta = M_{\alpha+\beta}$
 - $M_\alpha R_\beta = M_{\alpha-\beta}$
 - $M_\alpha M_\beta = R_{\alpha-\beta}$

Classify:

- identity - rotation
- reflection
- translation - glide

How to show these?

Multiply the relevant matrices.

$$\cos(\theta \pm \varphi) = \cos \theta \cos \varphi \pm \sin \theta \sin \varphi$$

$$\sin(\theta \pm \varphi) = \sin \theta \cos \varphi \pm \cos \theta \sin \varphi$$