I : id

$R_{a,\theta}$: rotation about $\bar a$ via angle θ.

T_a : translation by vector $\bar a$.

$M_{a,b}$: reflection in the line joining $\bar a$ to $\bar b$.

$G_{a,b}$: glide in the line joining $\bar a$ to $\bar b$, that takes $\bar a$ to $\bar b$

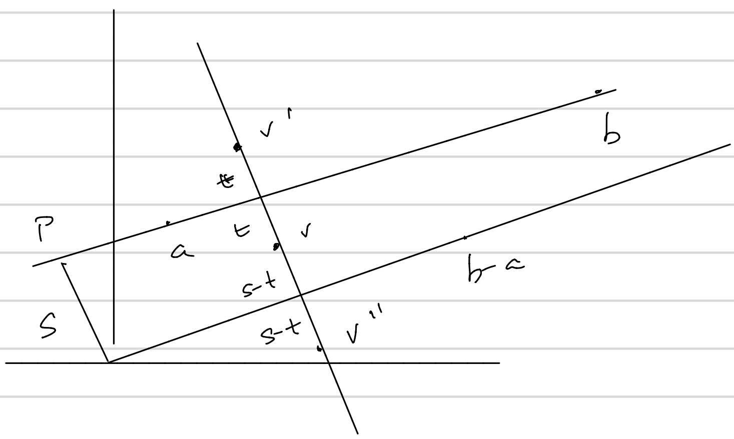
Thm: Let $\bar p$ be the foot of the perpendicular from $\bar 0$ to the line joining $\bar a$ to $\bar b$.

(1) $T_a(\bar v) = \bar v + \bar a$

(2) $R_{a,\theta}(\bar v) = R_\theta(\bar v - \bar a) + \bar a$

(3) $M_{\bar a,\bar b}(\bar v) = M_{\bar b - \bar a}(\bar v) + 2\bar p$.

(4) $G_{a,b}(\bar v) = M_{\bar b - \bar a}(\bar v) + 2\bar p + \bar b - \bar a$.



(3) : Easy to see:

$\vec v' = \vec v'' + 2\vec p$

where $\vec v'$ is the reflection of

$\vec v$: $\vec v' = M_{\bar a,\bar b}(\vec v)$

and $\vec v'' = M_{\bar b - \bar a}(\vec v)$.

(4) obvious from (3).

Thm: Every plane isometry is a product of at most 3 reflections.