Now given $z = x+iy$, how do we find $P = (X, Y, Z)$?

Well, P must be the intersection of (e) with the sphere $x^2+y^2+z^2=1$.

Since P is on (e):
$$\begin{cases} X = tx \\ Y = ty \\ Z = 1-t \end{cases}$$

Now, plug this in $x^2+y^2+z^2=1$ to find t.

$$(tx)^2 + (ty)^2 + (1-t)^2 = 1$$

$$\rightsquigarrow t^2(x^2+y^2) + t^2 - 2t = 0$$

$$\rightsquigarrow t\left[t(x^2+y^2) + t - 2\right) = 0$$

$$\rightsquigarrow \begin{cases} t = 0 \\ t = \dfrac{2}{x^2+y^2+1} \end{cases}$$

$$t=0 \rightsquigarrow (X, Y, Z) = (0, 0, 1)$$

$$t = \frac{2}{x^2+y^2+1} \rightsquigarrow (X, Y, Z) = \left(\frac{2x}{x^2+y^2+1}, \frac{2y}{x^2+y^2+1}, \frac{x^2+y^2-1}{x^2+y^2+1}\right)$$

Since we start with a point $z \in \mathbb{C}$ ($\text{so } z \neq \infty$) the point $P = \left(\dfrac{2x}{x^2+y^2+1}, \dfrac{2y}{x^2+y^2+1}, \dfrac{x^2+y^2-1}{x^2+y^2+1}\right)$.