$A: n \times n$ is an <u>orthogonal matrix</u> if columns of A are orthonormal.

$\det(A) = \det(A^T) = \pm 1$

$(A^T A = I = AA^T)$

<u>Review</u>: matrix for a linear transformation

<u>Thm</u>: $f: \mathbb{R}^n \to \mathbb{R}^n$ is an central isometry iff A is orthogonal
$f(\vec{x}) = A\vec{x}$

<u>Proof</u>: WLOG, assume $n=2$.

$(\Rightarrow)$ $\vec{e_1}, \vec{e_2} \rightsquigarrow f(\vec{e_1}), f(\vec{e_2})$ orthonormal.

$\Rightarrow A$ is orthogonal.

$(\Leftarrow)$ Sps A is orthogonal. Let $\vec{u}, \vec{v} \in \mathbb{R}^2$.

Let $\vec{w} = \vec{u} - \vec{v}$.

$|A\vec{w}|^2 = (A\vec{w})^T (A\vec{w}) = \vec{w}^T A^T A \vec{w}$

$= \vec{w}^T \vec{w} = |\vec{w}|^2$

$\Rightarrow$ So $|A\vec{w}| = |\vec{w}|$ ☒

<u>Central Isometries</u>:

<u>Thm 1</u>: The matrix for ρ_θ: rotation by angle θ is

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Find eigenvalues/eigenvectors of R_θ

<u>Proof</u>: Let $P = (x, y)$ $x = r\cos\varphi$, $y = r\sin\varphi$