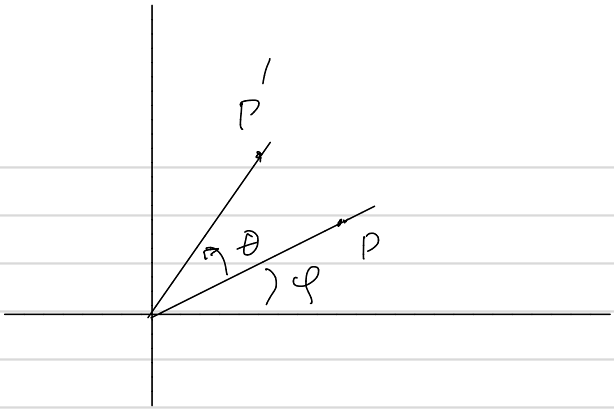

let $\rho_\theta(P) = P'$
$= (X, Y)$

$\to X = r\cos(\varphi + \theta)$
$Y = r\sin(\varphi + \theta)$

So $X = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta$
$= x\cos\theta - y\sin\theta$

$Y = x\cos\theta + y\sin\theta$

So $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Thm: The matrix for μ_θ is $M_{2\theta} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Proof: Ex. — reflection along line (thru 0 w/ angle θ relative to positive x-axis

Thm: Every central isometry of the plane is either a rotation about the origin or a reflection in a line thru the origin.

Proof: $f: \mathbb{R}^2 \to \mathbb{R}^2$ isometry $\to f(\vec{x}) = A\vec{x}$ f.s. A

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ orthogonal

$\leadsto \begin{cases} a^2 + c^2 = 1 & \to a = \cos\theta, \ c = \sin\theta \\ b^2 + d^2 = 1 & \to b = \cos\varphi, \ d = \sin\varphi \\ ab + cd = 0 & \to \cos\varphi\cos\theta + \sin\varphi\sin\theta = 0 \end{cases}$