HOMEWORK 3: ITERATIVE METHODS FOR FEM

DUE DEC 5

(1) The product of two symmetric matrices is not necessarily symmetric but will be in a different inner product.
   (a) Suppose A and B are SPD, prove that BA is SPD with respect to the inner product $(\cdot\,,\cdot)_A$ or $(\cdot\,,\cdot)_{B^{-1}}$.
   (b) Prove that
$$I - \bar{B}A = (I - BA)^*(I - BA).$$
(2) Prove that the convergence rate of Richardson, weighted Jacobi method, and Gauss-Seidal method for the 5-point stencil finite difference method of the Poisson equation on a uniform mesh with size h, is like
$$\rho \leq 1 - Ch^2.$$
Thus when $h \to 0$, we will observe slow convergence of those classical iterative methods.
(3) Let us consider the matrix equation
$$Au = f,$$
where A is an $N \times N$ SPD matrix. Let us take the trivial decomposition of $\mathbb{R}^N = \sum_{i=1}^N \operatorname{span}\{e_i\}$, where $\{e_i, i = 1, \ldots, N\}$ is the canonical basis of $\mathbb{R}^N$. Prove that
   - for $R_i = \omega I$, PSC is Richardson method;
   - for $R_i = A_i^{-1}$, PSC is Jacobi method;
   - for $R_i = A_i^{-1}$, SSC is the Gauss-Seidal method.
(4) Prove that PSC using local solvers $R_i = A_i^{-1}$ is equivalent to the Jacobi method for solving the large system $\tilde{A}\tilde{u} = \tilde{f}$.

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