PROJECT 3: CONFORMING AND NON-CONFORMING FINITE ELEMENT METHODS FOR LINEAR ELASTICITY

We shall consider finite element methods for solving the plane elasticity problem

$$-\mu\Delta \boldsymbol{u} - (\mu+\lambda)\,\mathrm{grad}(\mathrm{div}\,\boldsymbol{u}) = \boldsymbol{f} \quad \text{in } \Omega, \tag{1}$$

$$\boldsymbol{u} = \boldsymbol{g} \quad \text{on } \partial\Omega. \tag{2}$$

The weak formulation of (1)-(2) is: Find $\boldsymbol{u} \in \boldsymbol{H}_g^1(\Omega)$ such that

$$\int_\Omega \mu\nabla \boldsymbol{u} : \nabla \boldsymbol{v} + (\mu+\lambda)\,\mathrm{div}\,\boldsymbol{u}\,\mathrm{div}\,\boldsymbol{v}\,d\boldsymbol{x} = \int_\Omega \boldsymbol{f}\boldsymbol{v}\,d\boldsymbol{x}, \tag{3}$$

for all $\boldsymbol{v} \in \boldsymbol{H}_0^1(\Omega)$.

(1) Implement conforming $\mathcal{P}_1$ element approximation and show the locking when $\lambda \to \infty$.

(2) Implement non-conforming $\mathcal{P}_1$ element approximation and show the locking-free convergence i.e. uniform convergence with respect to λ.

(3) *(Optional) Implement conforming $\mathcal{P}_2$ element approximation and show the locking-free convergence i.e. uniform convergence with respect to λ.

Test examples: you can choose smooth solution and plug into the equation to get the corresponding right hand side and boundary condition.

Then using your code to compute the problem in the paper (page 12)

Wihler, T.P. Locking-free adaptive discontinuous Galerkin FEM for linear elasticity problems. Mathematics of Computation, 75:1087, 2006.