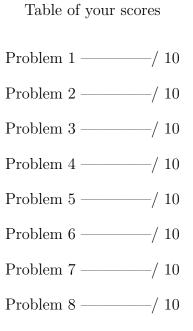
Print Your Name: —	last	first
Print Your I.D. Numb	er:	

Qualifying Examination/ Complex Analysis

September, 2009



Total ———/ 80

Notation: Let $D(z_0, r)$ be the disk centered at z_0 and radius r in the complex plane **C**.

1. For $\alpha, \beta, \gamma > 0$, find the radius of convergence for the series

$$\sum_{n=0}^{\infty} \frac{\alpha \left(\alpha+1\right) \dots \left(\alpha+n-1\right) \beta \left(\beta+1\right) \dots \left(\beta+n-1\right)}{n! \gamma \left(\gamma+1\right) \dots \left(\gamma+n-1\right)} z^{n}.$$

2. Prove or disprove there is a holomorphic function f(z) on the unit disc D(0,1) so that

$$\left\{z \in D(0,1) : f^{(k)}(z) = 0 \text{ for some non-negative integer } k\right\} = (-1,1)$$

where $f^{(k)}$ is the *k*th derivative of *f*.

3. Let $L \subset \mathbf{C}$ be the line $L = \{z = x + iy \mid y = 2\}$. Assume that $f : \mathbf{C} \to \mathbf{C}$ is an entire function such that for any $z \in L$ we have $f(z) \in L$. Assume that f(0) = i. Find f(4i).

4. Let $\{f_{\alpha}\}_{\alpha \in A}$ be a family of holomorphic functions on the unit disc D(0, 1) such that

for all
$$z \in D(0,1)$$
 $\forall f \in \{f_{\alpha}\}_{\alpha \in A}$ $\operatorname{Im} f(z) \neq (\operatorname{Re} f(z))^2$.

Prove that $\{f_{\alpha}\}_{\alpha \in A}$ is a normal family (i.e. every sequence in $\{f_{\alpha}\}_{\alpha \in A}$ has a subsequence that converges or tends to infinity uniformly on compact subsets of D(0, 1)).

5. Let $f(z) : \mathbf{C} \setminus \{0,1\} \to \mathbf{C} \setminus D(0,1)$ be holomorphic. Prove that f must be a constant.

6. Suppose that f is a polynomial such that all of its zeros are inside of the unit disc. Prove that all zeros of f' are also inside of the unit disc.

7. Find the integral

$$\int_0^\infty \frac{1 - \cos(2x)}{x^2} dx$$

8. Does there exist a sequence of holomorphic functions $\{f_n(z)\}_{n=1}^{\infty}$ on the unit disc D(0,1) so that $f_n(z) \to 1/z$ uniformly on $\{z \in \mathbf{C} : |z| = 1/2\}$ as $n \to \infty$?