MATH 2B: SAMPLE FINAL #1

- This exam consists of 14 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.
- 1. (6 points) Suppose that $\int_{-1}^{1} f(x) dx = 6$, $\int_{1}^{4} f(x) dx = -2$ and $\int_{-1}^{1} h(x) dx = 9$. Use this information to compute the following.

a.
$$\int_{4}^{1} 6f(x) dx$$

b.
$$\int_{-1}^{1} [2f(x) + 3h(x)] dx$$

c.
$$\int_{-1}^{4} f(x) dx$$

- 2. (6 points)
 - a. Evaluate the following derivative

$$\frac{d}{dx} \int_{\sin(x)}^{x^2} t^3 \tan(t) \, dt.$$

b. Let r(t) be the rate at which the world's oil is consumed, where t is measured in years starting at t=0 representing January 1, 2000, and r(t) is measured in barrels per year. What does $\int_0^{13} r(t) \, dt$ represent and what are its units?

3. (6 points) Evaluate $\int x^2 \tan^{-1} x \, dx$

4. (6 points) Evaluate $\int \frac{1}{x \ln(3x)} dx$

5. (6 points) Evaluate $\int \sin^5(x) \cos^2(x) dx$

6. (6 points) Evaluate $\int \frac{\sqrt{x^2 - 25}}{x} dx$, where x > 5

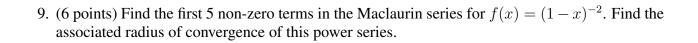
7. (6 points) Determine whether each of the following improper integrals are convergent or divergent. Evaluate the integral if it is convergent.

a.
$$\int_0^2 \frac{1}{(x-2)^2} dx$$

b.
$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

- 8. (6 points)
 - a. Find the average value of the function $f(x) = \sec^2(x)$ on the interval $[0, \frac{\pi}{4}]$.

b. Find the arc length of the curve given by $y = 2x^{3/2}$ from x = 0 to x = 1.



10. (6 points) Determine whether each of the following sequences converges or diverges. If it converges, find the limit.

a.
$$a_n = (\frac{2}{3})^n + 3$$

b.
$$b_n = n^3 e^{-n}$$

c.
$$c_n = \tan^{-1}(\ln(n))$$

11. (10 points) Find the area of the region(s) bounded by the curves $y=x^3$ and y=4x.

12. (10 points)

a. The region bounded by the curve $y = x^2 + 1$ and the line y = -x + 3 is revolved about the line y = 5 to generate a solid. Find the volume of that solid.

b. Let R be the region bounded by the curve $y=x^2+1$ and the line y=-x+3. Find the volume of the solid with base R and cross-sections perpendicular to the x-axis are squares.

- 13. (10 points) Answer True or False to each of the following and briefly explain your answers.
 - a. True/False: We have $\int_0^5 |x^2 3x 4| dx \ge 0.$

b. True/False: We have $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \pi^{2k} = -1$.

c. True/False: We have

$$\frac{d}{dx}\left(\int_0^{\pi/4}\cos(x)\,dx\right) = \frac{\sqrt{2}-2}{2}.$$

d. True/False: There is a positive integer m such that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m-1} + \frac{1}{m} > 20$.

14. (10 points) Determine whether each of the following series is convergent or divergent. Indicate test used.

a.
$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$$

b.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

$$c. \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

d.
$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$e. \sum_{n=1}^{\infty} \frac{1}{3^n - 1}$$