MATH 2B: SAMPLE FINAL #2

- This exam consists of 13 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (8 points) Consider continuous functions f and f' (where f' denotes the derivative of f) with values given by the following table:

x	0	1	2	3	4	5
$f(x)$	3	4	6	9	13	18
$f'(x)$	1	2	4	6	7	5

   a. Find $\int_0^4 f'(x)\,dx$

   b. Estimate $\int_1^4 f(x)\,dx$ Using a left-hand Riemann sum with 3 equal subintervals.

   c. Evaluate the following derivative at the point $x = 3$

$$\frac{d}{dx}\left(\int_2^x f(t)\,dt\right)$$

   d. Suppose $f(x)$ gives the height of a rocket, measured in yards, x minutes after its launch. What are the units of $\int_0^4 f'(x)\,dx$ and what does this quantity represent.

2. (7 points) Evaluate $\displaystyle\int \frac{x}{1+x^4}\,dx$

3. (7 points) Evaluate $\int \frac{x^2}{e^{2x}}\, dx$

4. (7 points) Evaluate $\int \sin^3(4t)\, dt$

5. (7 points) Evaluate the following integral by making an appropriate trigonometric substitution.

$$\int \frac{dx}{x^2\sqrt{x^2-9}}$$

6. (8 points) Determine whether the following integral is convergent or divergent. Evaluate the integral if it convergent. If it is divergent, explain why.

$$\int_0^\infty \frac{dz}{z^2+3z+2}$$

7. (10 points) Find the area of the region bounded by the curves $y = \frac{3}{2} - \frac{x^2}{2}$ and $y = |x|$.

8. (10 points) Find the volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = \sqrt{4 - x^2}$ and $y = 2 - x$.

9. (6 points) Determine whether each of the following sequences is convergent or divergent. Find the limit of the convergent sequences.

   a. $a_n = \dfrac{e^{2n}}{\sqrt{n}}$

   b. $a_n = \dfrac{(-1)^n}{n!}$

   c. $a_n = \tan^{-1}(n)$

10. (6 points) Compute the arc length of the curve $y = \ln(\cos(x))$ over the interval $[0, \frac{\pi}{4}]$. (Hint. $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$.)

11. (12 points) Use the indicated test to determine whether the given series is convergent or divergent.

a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ (integral test)

b. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ (ratio test)

c. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+5}$ (alternating series test)

d. $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1}$ (comparison test or limit comparison test)

12. (6 points) Find the sum of the following convergent series.

a. $\sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}}$

b. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

13. (6 points) Find a power series representation for the function $f(x) = \frac{2}{3-x}$ and determine the interval of convergence.