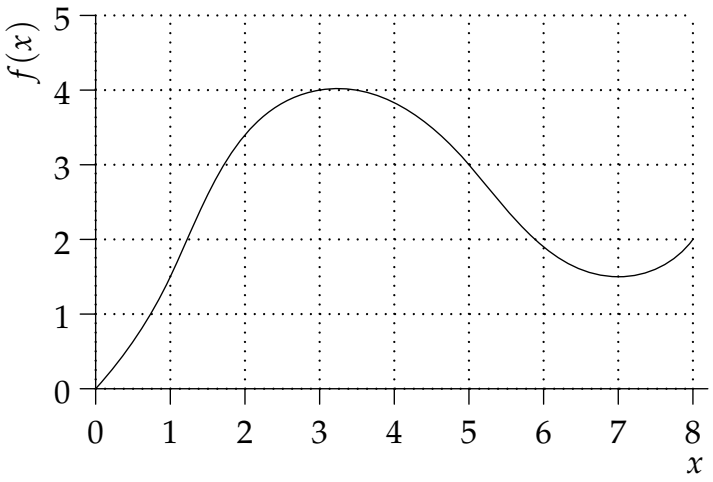
Math 2B: Sample Final 3

- Turn off your cell phone and do not check it during the exam.
- No calculators or other forms of assistance allowed.
- This exam consists of 12 questions for 100 total points. Points per question are in brackets.
- Read the directions for each problem carefully and answer all parts of each problem.
- Unless instructed otherwise, show all work for full credit.
- Define any notation used and label any sketches/graphs.

1. For the function drawn, estimate the area under the curve using a Riemann sum with four subintervals and midpoints. Sketch the Riemann sum by drawing rectangles on the picture. (5)



2. Evaluate the following integrals

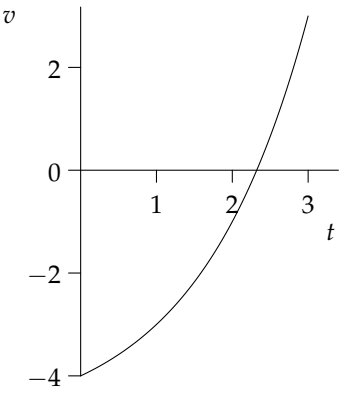
(a) $\int \sin\theta \cos^2\theta \, \mathrm{d}\theta$ (3)

(b) $\int \frac{x-7}{(x+1)(x-3)} \, \mathrm{d}x$ (8)

3. Compute the average value f_{av} of the function $f(x) = x \cos x$ on the interval $[0, \frac{\pi}{2}]$. (7)

4. A particle has velocity $v(t) = 2^t - 5$ ft/s at time t seconds.

   (a) Compute the *displacement* of the particle over the time interval $t = 0$ to $t = 3$. (4)



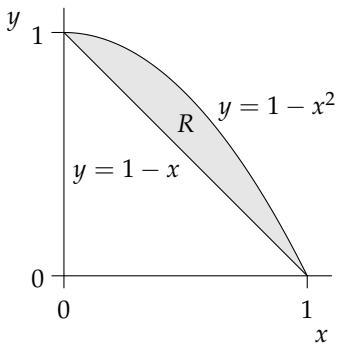
   (b) The *distance* travelled by the particle over the same time interval is given by

$$\int_0^a \left(5 - 2^t\right) \mathrm{d}t + \int_a^3 \left(2^t - 5\right) \mathrm{d}t$$

   where a is a constant. What is the value of a? (2)

5. A region R is drawn.

(a) Find the area of R. (4)



(b) Set up, but don't compute, an integral for the volume when R is rotated around the line $x = -2$. (5)

6. Compute the integral $\int \frac{9}{(9+x^2)^{3/2}}\,\mathrm{d}x$ (8)

7. You are given the following information about a function f.

$$f(0) = 5, \qquad f(2) = 3, \qquad \int_0^2 f(x)\,\mathrm{d}x = -1$$

Compute the following:
(*Hint: You may find it convenient to use techniques like substitution or integration by parts*)

(a) $\displaystyle\int_0^2 \big(3f(x) + 4\big)\,\mathrm{d}x$ (2)

(b) $\displaystyle\int_0^1 f(2x)\,\mathrm{d}x$ (4)

(c) $\displaystyle\int_0^2 xf'(x)\,\mathrm{d}x$ (4)

8. Decide whether each of the following series converges or diverges. Make sure you state which test you are using.

(a) $\sum_{n=1}^{\infty}(-1)^n\left(3+\frac{1}{n}\right)$ (3)

(b) $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n+1}\right)^n$ (4)

(c) $\sum_{n=1}^{\infty}\frac{1}{n^2+1+n^{-1}}$ (4)

9. Find the interval of convergence of the power series (8)

$$\sum_{n=2}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n}}$$

10. Compute the first three terms of the Taylor series of the function $f(x) = \sqrt{x}$ centered at $x = 4$.

(6)

11. Answer *true* or *false* to each of the following and give a short explanation. Unjustified answers will receive no credit.

(a) $\int_{-2}^{2} \sin(x^3) + x^2 \, dx = \frac{16}{3}$ (3)

(b) $\int_{0}^{2} \frac{1}{x-1} \, dx = 0$ (3)

(c) $\frac{d}{dx} \int_{3}^{x^2} \cos(t^2) \, dt = \cos(x^4) - \cos 9$ (3)

(d) The function $f(x) = \frac{4}{4+x^2}$ may be represented by the power series $\sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n x^{2n}$. (4)

12. Define a sequence $(x_n)_{n=1}^{\infty}$ as follows

$$\begin{cases} x_1 = 1 \\ x_{n+1} = \dfrac{n^2}{n^2+1} x_n \end{cases}$$

We therefore have $x_2 = \dfrac{1^2}{1^2+1} x_1 = \dfrac{1}{2}$.

(a) Compute x_3 and x_4. (2)

(b) Use a Theorem to show that the sequence (x_n) converges to a limit. (4)
(*Do not try to compute the limit!*)