Math 2B: Midterm Sample

This exam consists of 9 questions and 100 total points. The point value of each problem is indicated. Read directions for each problem carefully. Please show all work needed to arrive at your solutions. Clearly indicate your final answers.

1) (5 points each) Integrate.

$a)\ \int \frac{\ln t}{t^5}\, dt$

$b)\ \int \sin^5\theta\ d\theta$

c) $\int \frac{dx}{\left(4-x^2\right)^{3/2}}$

d) $\int \frac{dx}{\sqrt{1-x^2}\, sin^{-1}x}$

2) (5 points each) Determine whether each improper integral below is convergent or divergent. Evaluate those that are convergent.

a) $\displaystyle\int_{2}^{\infty} \frac{dx}{\sqrt{x}}$

b) $\displaystyle\int_{0}^{3} \frac{1}{\sqrt{9 - x^2}}\, dx$

c) $\displaystyle\int_{0}^{\infty} \frac{dz}{z^2 + 3z + 2}$

3. a) (5 points) Estimate, using Riemann sums, the area under the graph of $f(x) = x^2$ and above the x-axis from $x = 1$ to $x = 4$ by using three approximating rectangles and right-hand endpoints.

b) (10 points) Find $\int_1^4 x^2 dx$ by using the definition of the definite integral.

4. (5 points) Find $g'(x)$ if $g(x) = \int_{\sqrt{x}}^{1} \theta \cos \theta d\theta$.

5. a) (5 points) Find the area of the region bounded by $y = \sqrt{x-1}$, $y = 0$, and $x = 5$.

b) (10 points) Find the volume of the solid generated by revolving the region described in part a about $y = 3$.

c) (10 points) Find the volume of the solid whose is base is the region described in part a and whose cross-sections perpendicular to the y-axis are squares with side lying in the xy-plane.

6. (5 points) Find the average value of the function $f(x) = tan^3 x \cdot sec^2 x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

7. (5 points) If f is continuous for all real numbers and $\int_1^3 f(x)dx = 4$, find $\int_1^9 \frac{f(\sqrt{x})}{\sqrt{x}} dx$

8. (5 points) A particle moves along a line so that its velocity at time t is $v(t) = |2 - t|$. Find the displacement of the particle during the time period $0 \le t \le 3$.

9. (5 points) Below is the graph of the function $v(t)$. Let $g(x) = \int_0^x v(t)dt$.

Find $g(2)$, $g(3)$, $g'(0)$, $g'(2)$, $g''(5)$.